Fully Homomorphic Encryption for Matrix Arithmetic

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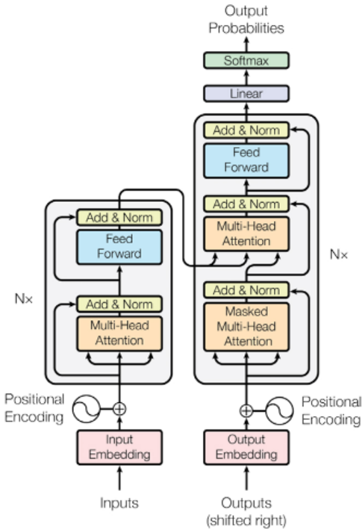
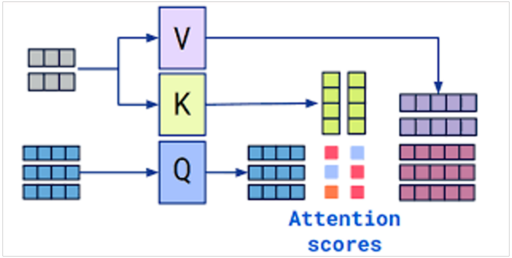
Mar. 8, 2026

Motivation: Privacy-Preserving LLM





Goal

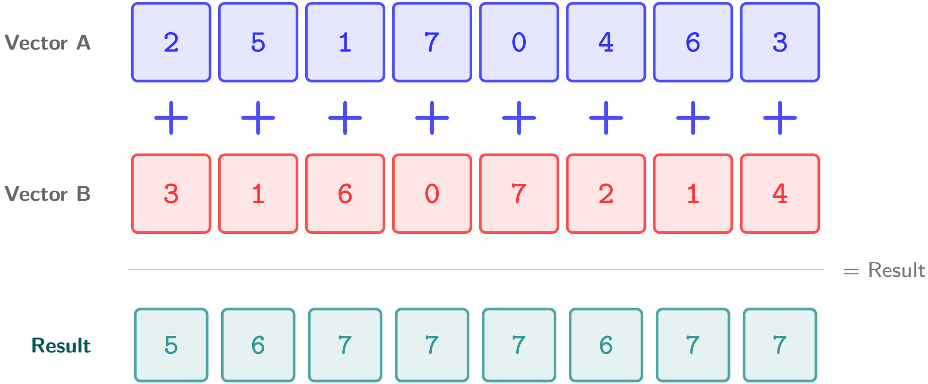
1. Ct-Ct Matrix multiplication as an **atomic** operation
   - Reduced to nonencrypted matrix-matrix multiplication
   - Single key switching, not relying on rotations or slot–coefficient transformation
   - Flexible matrix size
2. Addition and Hadamard products as-is
3. All possible rotations and transpositions

CKKS Binary Operations: Addition

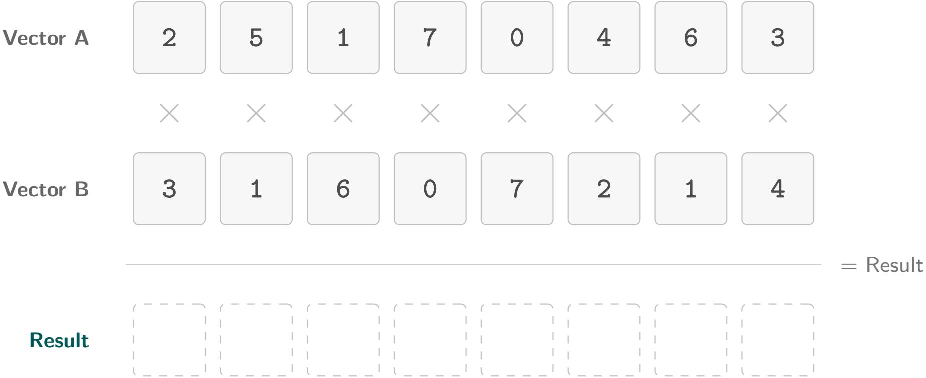
Vector A	2	5	1	7	0	4	6	3
	+	+	+	+	+	+	+	+
Vector B	3	1	6	0	7	2	1	4
Result								

= Result

CKKS Binary Operations: Addition



CKKS Binary Operations: Multiplication



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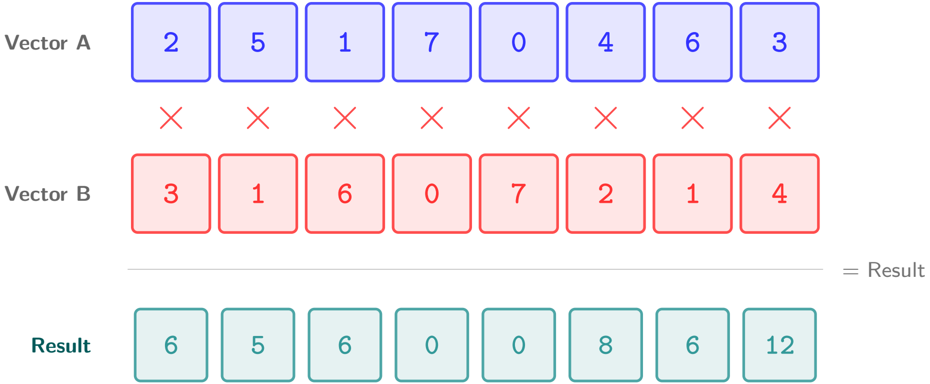
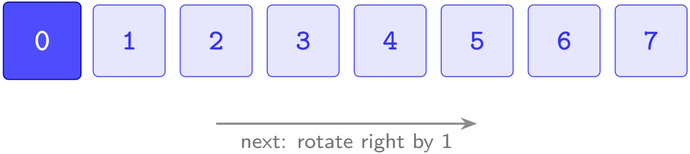


CKKS Rotations



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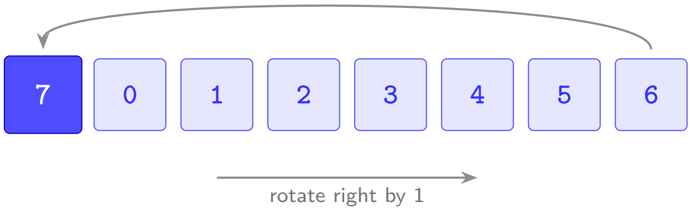
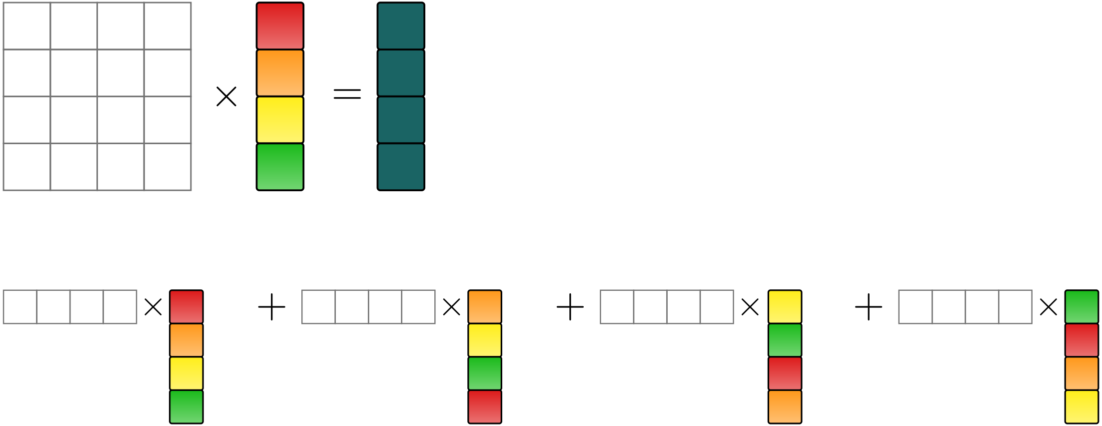


Matrix Multiplication with Rotation (Simplified)



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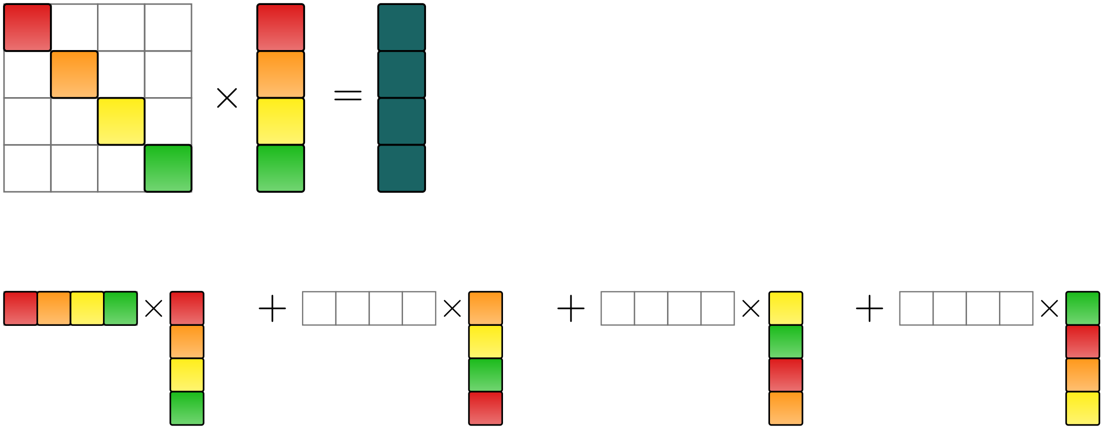

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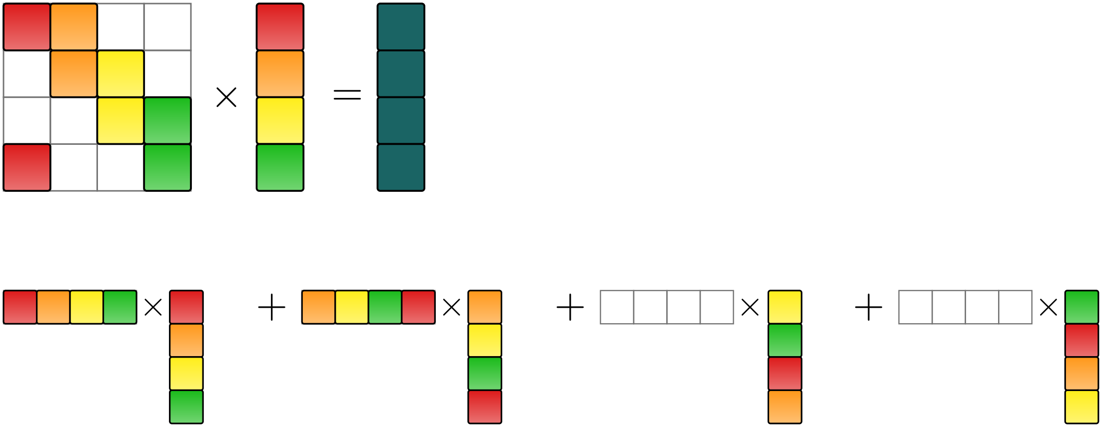

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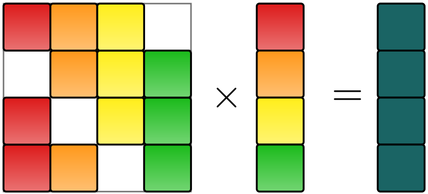

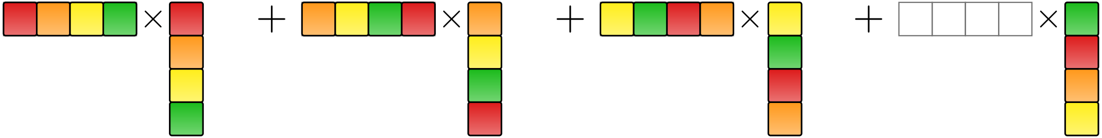

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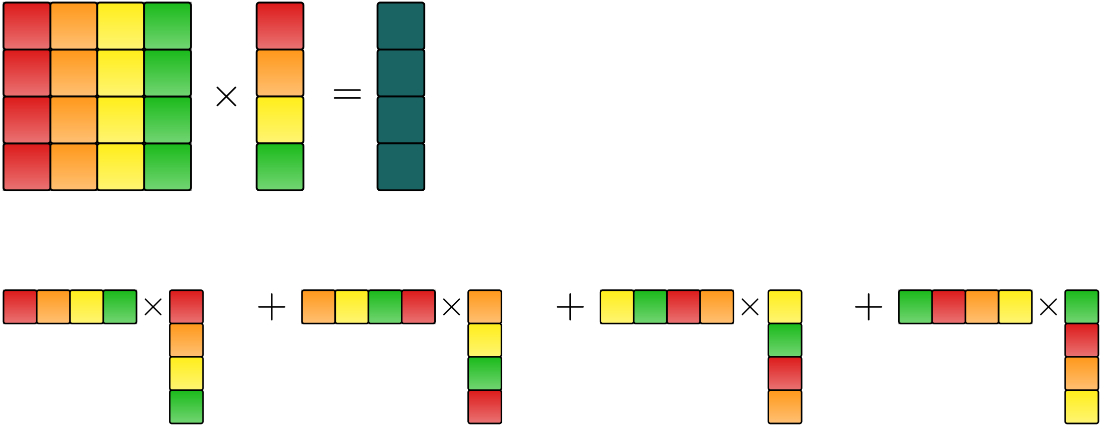


Important Related Works and Limitations

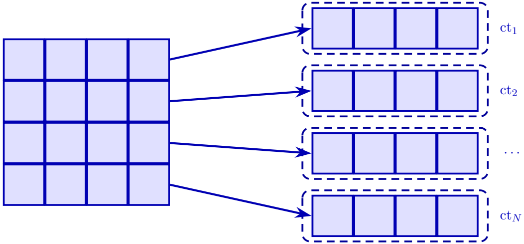
Pt-Ct matrix multiplication to Pt-Pt matrix multiplication:

- Liu, J., Zhang, L.F., *Privacy-preserving and publicly verifiable matrix multiplication,* IEEE Transactions on Services Computing, IEEE Tran. on Serv. Comput. 2023.
- Bae, Y., Cheon, J.H., Hanrot, G., Park, J.H., Stehlé, D., *Plaintext-ciphertext matrix multiplication and FHE bootstrapping,* CRYPTO'24.

Ct-Ct matrix multiplication to Pt-Pt matrix multiplication:

- Park, J.H., *Ciphertext-ciphertext matrix multiplication: Fast for large matrices,* Eurocrypt'25.
- Cheon, J.H., Kang, M., Lee, J., *Fast batch matrix multiplication in ciphertexts,* ia.cr/2025/1957 (parallel).

Important Related Works and Limitations (Contd.)



Limitations

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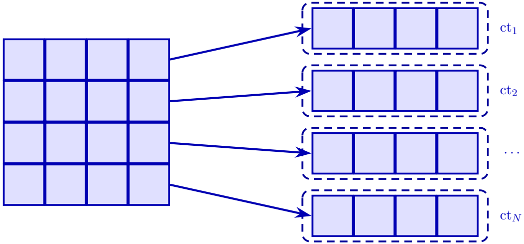

Limitations

- Matrix dimension $\approx$ coefficient count; ring switching required[1]

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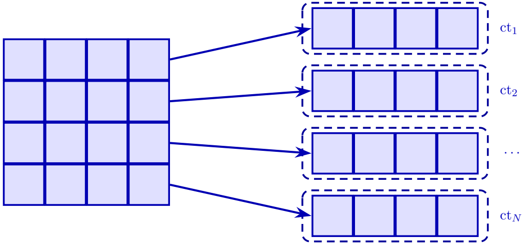

Limitations

- Matrix dimension $\approx$ coefficient count; ring switching required[1]
- Slot–coefficient transformation required

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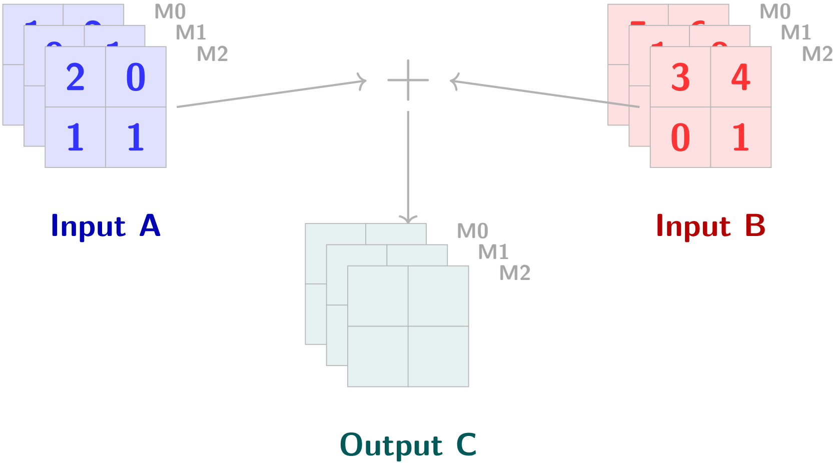
Proposed Binary Operations: Addition

Batch-wise: for each $m \in \{0, 1, 2\}$, $C_m = A_m + B_m$

- Two plain additions in $\mathbb{Z}_q$
- No key switching



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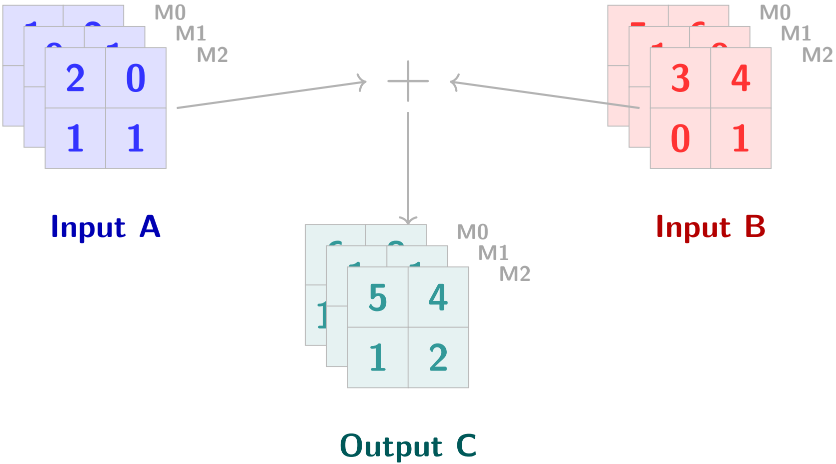
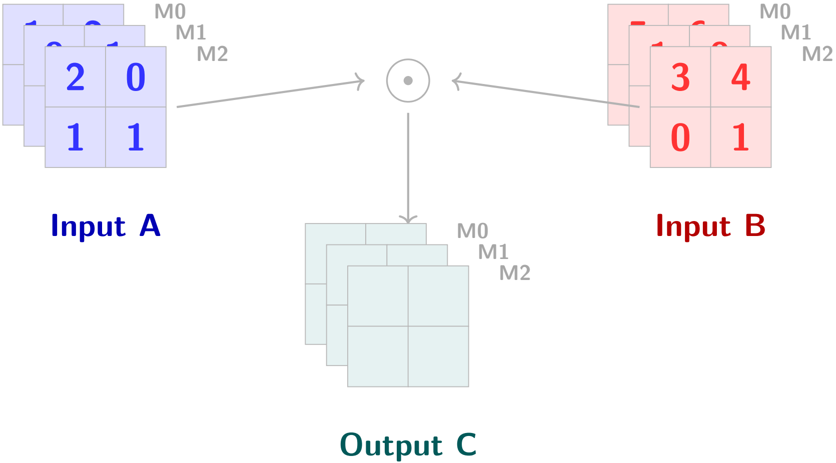


Proposed Binary Operations: Hadamard Multiplication

Batch-wise Hadamard: for each $m \in \{0, 1, 2\}$, $C_m = A_m \odot B_m$

- Four plain ring multiplications in $\mathbb{Z}_q$
- Single key switching.



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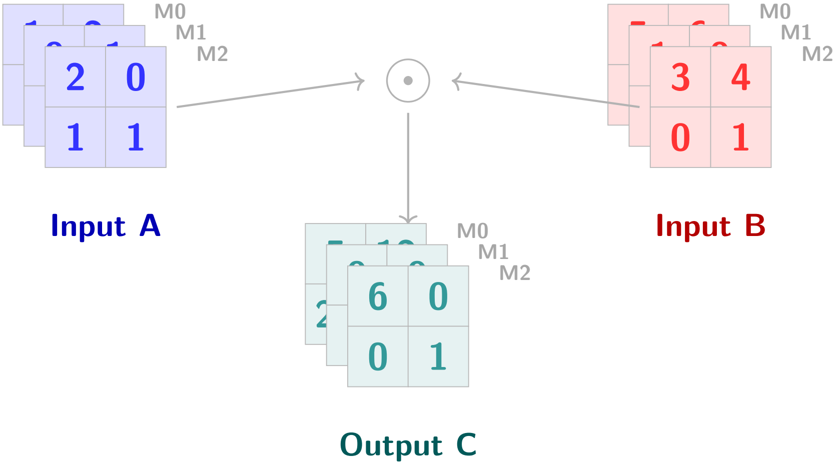
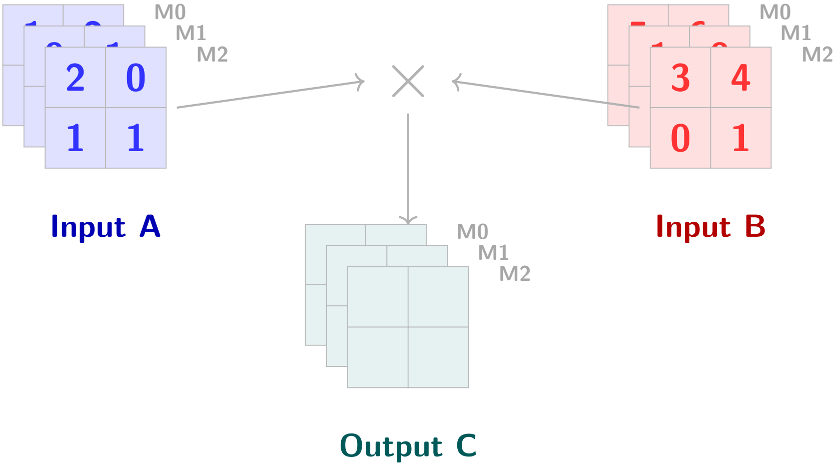


Proposed Binary Operations: Matrix Multiplication

Batch-wise matrix product: for each $m \in \{0,1,2\}$, $C_m = A_m B_m$

- Four plain MMs in $\mathbb{Z}_q[i]$
- Single (up to x4 heavy) key switching, no rotations or slot–coefficient transformation



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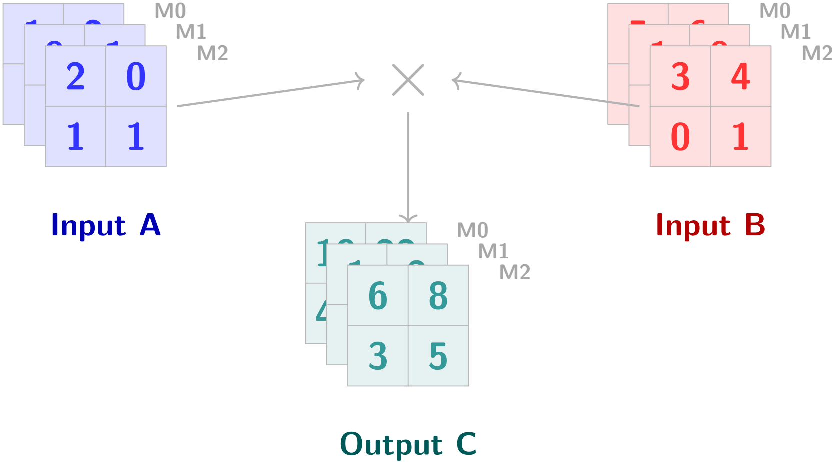
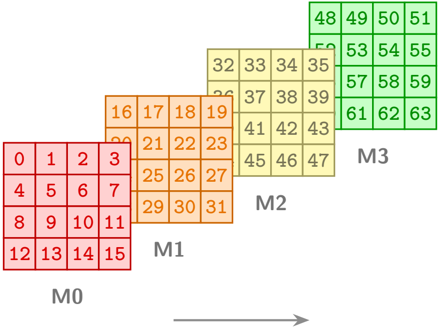
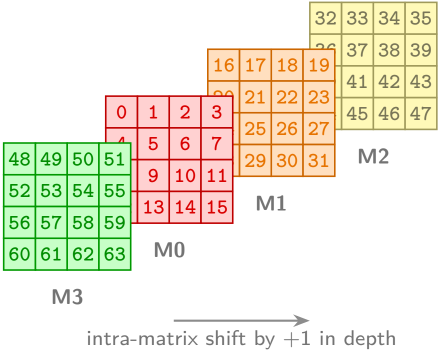


Inter-Matrix Rotations

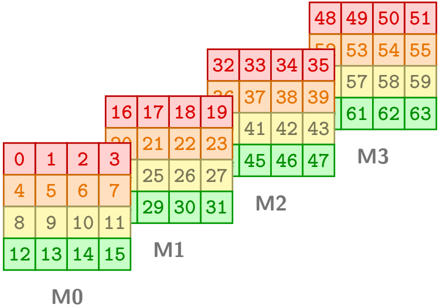


next: intra-matrix shift by +1 in depth

Inter-Matrix Rotations

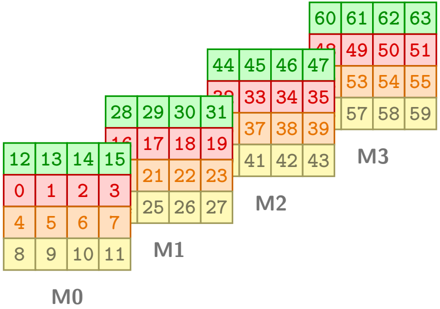


Intra-Matrix Rotations: Row



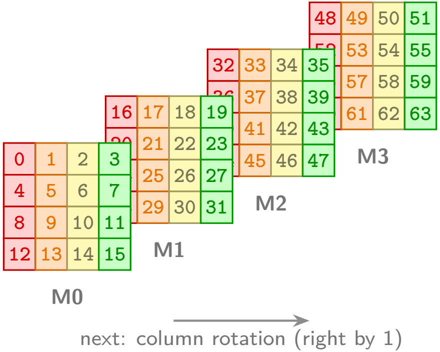
next: row rotation (down by 1)

Intra-Matrix Rotations: Row

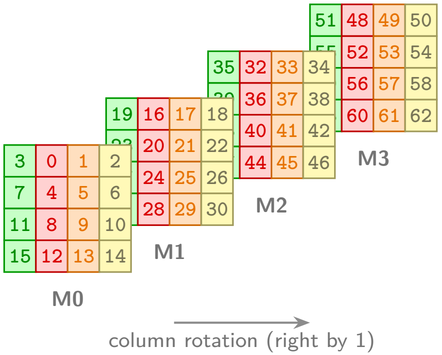


row rotation (down by 1)

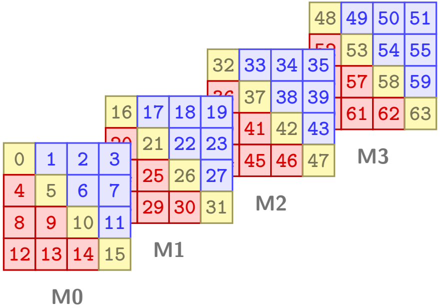
Intra-Matrix Rotations: Column



Intra-Matrix Rotations: Column

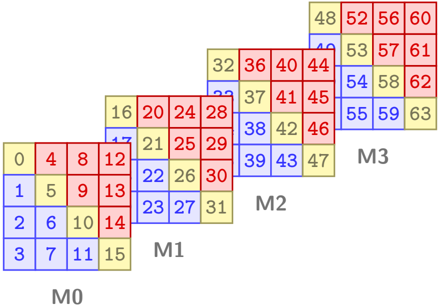


Matrix Transposition



next: transposition $(A \to A^{\mathsf{T}})$

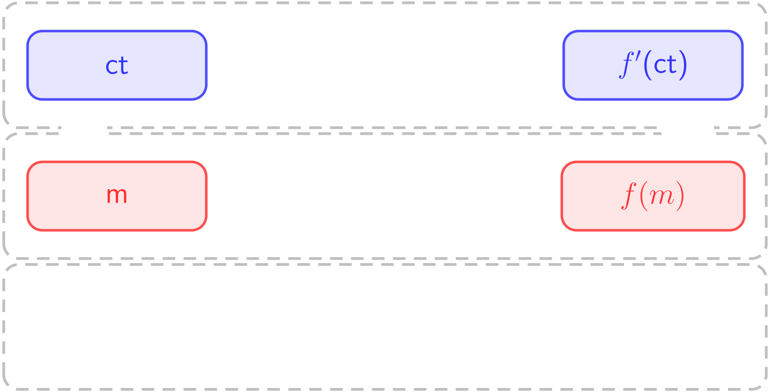
Matrix Transposition



transposition $(A \to A^{\mathsf{T}})$

Technical Details

Homomorphic Encryption



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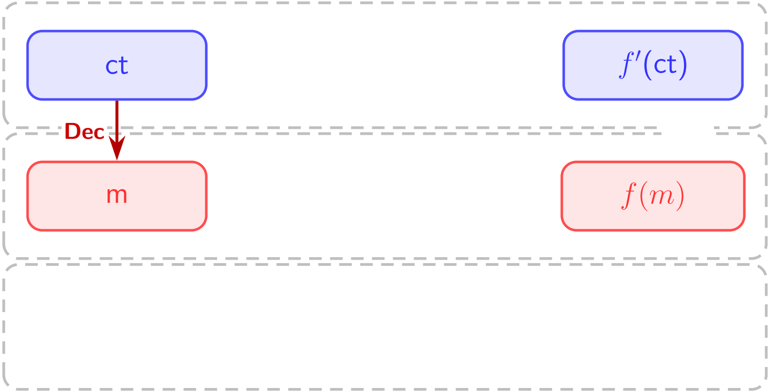

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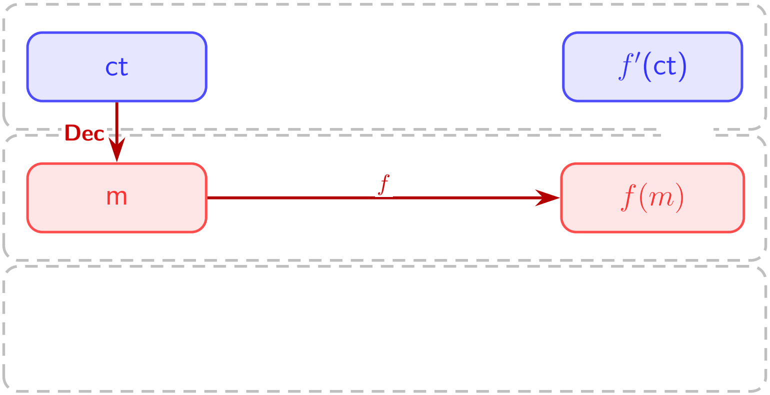

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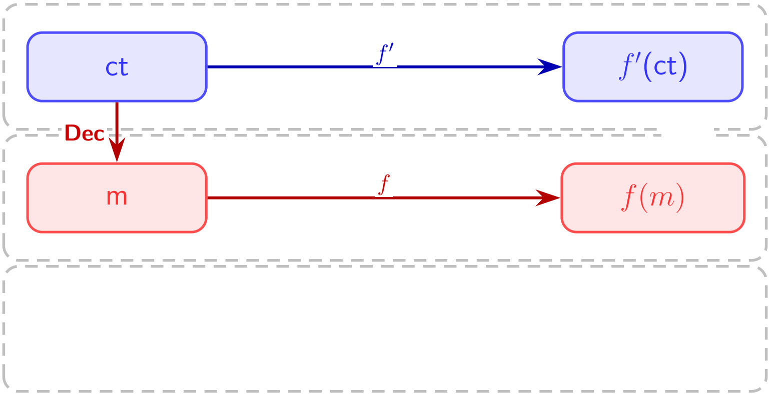

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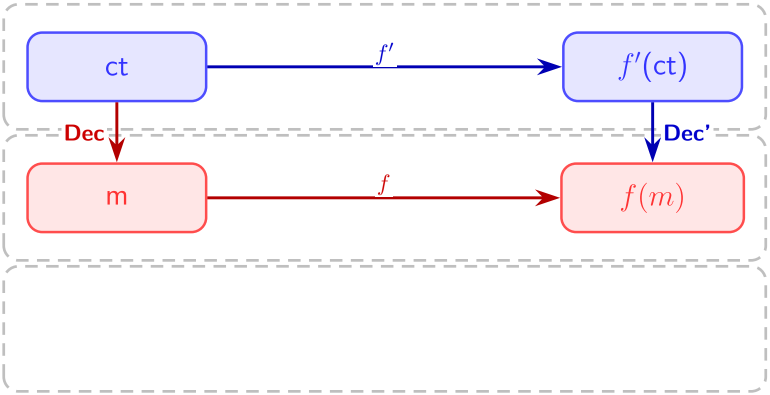

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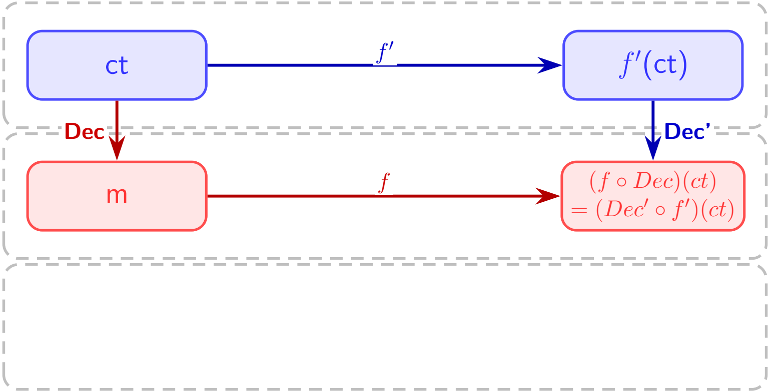
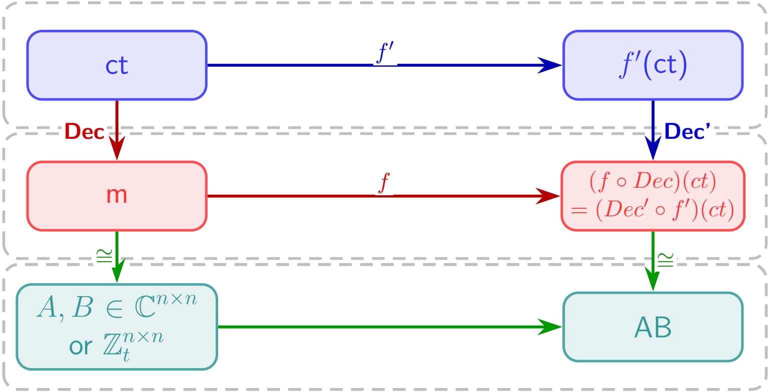


Decryption and f commute

Homomorphic Encryption



Decryption and f commute and f should be meaningful

Basic Characteristics of Trace

$$\mathrm{Tr} : S \to R, \quad s \mapsto \sum_{\sigma} \sigma(s)$$

S is some extension of R, and σ are automorphisms fixing R.

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S is some extension of R, and σ are automorphisms fixing R. Equivalently, $\mathrm{Tr}(s) = \sum_{\ell} s(Z^{\ell})$.

Linearity

- $\mathrm{Tr}(s_1 + s_2) = \mathrm{Tr}(s_1) + \mathrm{Tr}(s_2)$
- $\mathrm{Tr}(r \cdot s) = r \cdot \mathrm{Tr}(s)$ for $r \in R, s \in S$

Matrix Multiplication and Trace

Matrix Multiplication:

$$C_{jk} = \sum_{\ell} A_{j\ell} B_{\ell k}$$

Trace:

$$c(X, Y) = \sum_{\ell} a(X, Z^{\ell}) b(Y, Z^{\ell})$$

Our Decoding

Decoding:

$$a(\zeta_j, \zeta_k) = A_{jk},$$

$\zeta_j \in \mathbb{C}$ or $\mathbb{Z}_t$ are roots of unity.

Decoding of Traced Value:

$$c(X, Y) = \sum_Z a(X, Z) b(Y, Z)$$

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$$\begin{aligned} c(\zeta_j, \zeta_k) &= C_{jk} \\ &= \mathrm{Tr}(a(\zeta_j, Z) b(\zeta_k, Z)) \end{aligned}$$

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&= \sum_\ell a(\zeta_j, Z^\ell)b(\zeta_k, Z^\ell) \\
&= \sum_\ell a(\zeta_j, \zeta_\ell)b(\zeta_k, \zeta_\ell) = \sum_\ell A_{j\ell}B_{k\ell} \\
C_{jk} &= [AB^T]_{jk}
\end{aligned}$$

We have a meaningful f that commutes with ring addition and multiplication.

Encrypted Matrix Multiplication

Two ciphertexts under secret key $s(X)$ s is Y-free.

- $(b, a) : m(X, Y) = b(X, Y) + a(X, Y) \cdot s(X)$
- $(\beta, \alpha) : \mu(X, Y) = \beta(X, Y) + \alpha(X, Y) \cdot s(X)$

With permutations $(X, Y) \mapsto (X, Z)$ and $(X, Y) \mapsto (Y, Z)$:

- $m(X, Z) = b(X, Z) + a(X, Z) \cdot s(X)$
- $\mu(Y, Z) = \beta(Y, Z) + \alpha(Y, Z) \cdot s(Y)$

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- $\mu(Y, Z) = \beta(Y, Z) + \alpha(Y, Z) \cdot s(Y)$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$\mathrm{Tr}(m(X, Z) \cdot \mu(Y, Z))$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$
\begin{aligned}
&\mathrm{Tr}(m(X,Z)\cdot\mu(Y,Z)) \\
&= \mathrm{Tr}((b(X,Z)+a(X,Z)s(X))\cdot(\beta(Y,Z)+\alpha(Y,Z)s(Y)))
\end{aligned}
$$

Encrypted Matrix Multiplication (Contd.)

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&=\mathrm{Tr}(b(X,Z)\beta(Y,Z))+\mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z))
\end{aligned}
$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$
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&= \mathrm{Tr}((b(X,Z) + a(X,Z)s(X))\cdot(\beta(Y,Z) + \alpha(Y,Z)s(Y))) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + \mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z)) \\
&\quad + \mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y))
\end{aligned}
$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$
\begin{aligned}
&\mathrm{Tr}(m(X,Z)\cdot\mu(Y,Z)) \\
&= \mathrm{Tr}((b(X,Z)+a(X,Z)s(X))\cdot(\beta(Y,Z)+\alpha(Y,Z)s(Y))) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + \mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z)) \\
&\quad + \mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y)) + \mathrm{Tr}(a(X,Z)s(X)\alpha(Y,Z)s(Y))
\end{aligned}
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Encrypted Matrix Multiplication (Contd.)

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&=\mathrm{Tr}(b(X,Z)\beta(Y,Z))\ +\mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z))\\
&\quad+\mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y))\ +\mathrm{Tr}(a(X,Z)s(X)\alpha(Y,Z)s(Y))\\
&=\mathrm{Tr}(b(X,Z)\beta(Y,Z))+s(X)\mathrm{Tr}(a(X,Z)\beta(Y,Z))
\end{aligned}
$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$
\begin{aligned}
&\mathrm{Tr}(m(X,Z)\cdot\mu(Y,Z)) \\
&= \mathrm{Tr}((b(X,Z)+a(X,Z)s(X))\cdot(\beta(Y,Z)+\alpha(Y,Z)s(Y))) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + \mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z)) \\
&\quad + \mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y)) + \mathrm{Tr}(a(X,Z)s(X)\alpha(Y,Z)s(Y)) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + s(X)\mathrm{Tr}(a(X,Z)\beta(Y,Z)) \\
&\quad + s(Y)\mathrm{Tr}(b(X,Z)\alpha(Y,Z)) + s(X)s(Y)\mathrm{Tr}(a(X,Z)\alpha(Y,Z))
\end{aligned}
$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$
\begin{aligned}
&\mathrm{Tr}(m(X,Z)\cdot\mu(Y,Z)) \\
&= \mathrm{Tr}((b(X,Z)+a(X,Z)s(X))\cdot(\beta(Y,Z)+\alpha(Y,Z)s(Y))) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + \mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z)) \\
&\quad + \mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y)) + \mathrm{Tr}(a(X,Z)s(X)\alpha(Y,Z)s(Y)) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + s(X)\mathrm{Tr}(a(X,Z)\beta(Y,Z)) \\
&\quad + s(Y)\mathrm{Tr}(b(X,Z)\alpha(Y,Z)) + s(X)s(Y)\mathrm{Tr}(a(X,Z)\alpha(Y,Z))
\end{aligned}
$$

The ciphertext for the matrix multiplication is a 4-tuple:

$$\big(\mathrm{Tr}(b(X,Z)\beta(Y,Z))\,,\mathrm{Tr}(b(X,Z)\alpha(Y,Z))\,,\mathrm{Tr}(a(X,Z)\beta(Y,Z))\,,\mathrm{Tr}(a(X,Z)\alpha(Y,Z))\,\big),$$

Encrypted Matrix Multiplication (Contd.)

Trace of their product:

$$\begin{aligned}
&\mathrm{Tr}(m(X,Z)\cdot\mu(Y,Z)) \\
&= \mathrm{Tr}((b(X,Z)+a(X,Z)s(X))\cdot(\beta(Y,Z)+\alpha(Y,Z)s(Y))) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + \mathrm{Tr}(a(X,Z)s(X)\beta(Y,Z)) \\
&\quad + \mathrm{Tr}(b(X,Z)\alpha(Y,Z)s(Y)) + \mathrm{Tr}(a(X,Z)s(X)\alpha(Y,Z)s(Y)) \\
&= \mathrm{Tr}(b(X,Z)\beta(Y,Z)) + s(X)\mathrm{Tr}(a(X,Z)\beta(Y,Z)) \\
&\quad + s(Y)\mathrm{Tr}(b(X,Z)\alpha(Y,Z)) + s(X)s(Y)\mathrm{Tr}(a(X,Z)\alpha(Y,Z))
\end{aligned}$$

The ciphertext for the matrix multiplication is a 4-tuple:

$$\left(\mathrm{Tr}(b(X,Z)\beta(Y,Z)),\mathrm{Tr}(b(X,Z)\alpha(Y,Z)),\mathrm{Tr}(a(X,Z)\beta(Y,Z)),\mathrm{Tr}(a(X,Z)\alpha(Y,Z))\right),$$

whose decryption key is

$$(1, s(X), s(Y), s(X)s(Y)).$$

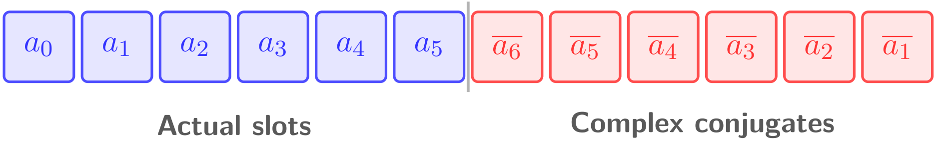
Tweak 1. Gaussian Integer: Removing Conjugate Symmetry

Simply extending to Y does not work:

Tweak 1. Gaussian Integer: Removing Conjugate Symmetry

Simply extending to Y does not work:
Conjugate symmetry is required in CKKS encoding.

$$\mathbb{Z}[X]/\langle X^n + 1 \rangle$$

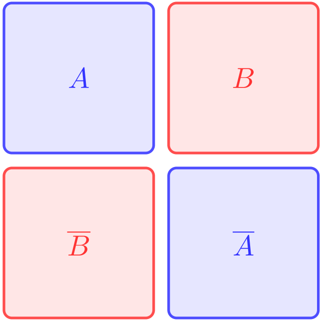


Tweak 1. Gaussian Integer: Removing Conjugate Symmetry

Simply extending to Y does not work:

- Fixed conjugate pairs
- Summing over all roots of unity mixes components

$$\mathbb{Z}[X, Y]/\langle X^n + 1, Y^n + 1\rangle$$



Tweak 1. Gaussian Integer: Removing Conjugate Symmetry

Our idea: remove conjugate symmetry using **Gaussian integers** $\mathbb{Z}[i]$.

- Simple isomorphism $\mathbb{Z}[i][X]/\langle X^{n/2} - i\rangle \cong \mathbb{Z}[X]/\langle X^n + 1\rangle$
- $\zeta_j = \zeta^{5^j}$ are roots of $X^{n/2} - i$, so we can decode $A_{jk} = a(\zeta_j, \zeta_k)$.

$$\mathbb{Z}[i][X,Y]/\langle X^{n/2} - i, Y^{n/2} - i\rangle$$

$$A \in \mathbb{C}^{n/2 \times n/2}$$

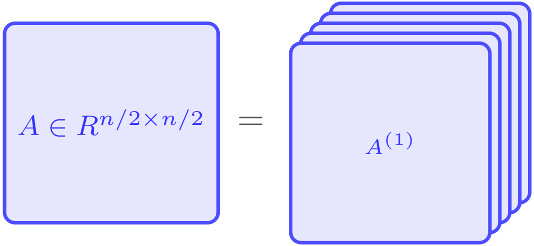
or $\in (\mathbb{Z}_t^{n/2 \times n/2})^2$

Tweak 2. Multi-Variate Ring: Flexible Matrix Dimensions

Remaining challenge: matrix dimensions must be $n/2$.

Our idea:

- Multivariate ring
  - $R[X,Y]/\langle X^{n/2}-i, Y^{n/2}-i\rangle \cong \mathbb{Z}[i][X,Y,W]/\langle X^{n/2}-i, Y^{n/2}-i, \Phi_p(W)\rangle$
- Matrix elements in $R = \mathbb{Z}[i][W]/\langle \Phi_p(W)\rangle$.
- Batching $\varphi(p)$ matrices in the W axis, much smaller n.



Security: Basic RLWE with Φ_{2np}

- Each ciphertext is based on ring $\mathbb{Z}[i][X,W]/\langle X^n - i, \Phi_p(W)\rangle$.
- For odd p, we have

$$\mathbb{Z}[i][X,W]/\langle X^n - i, \Phi_p(W)\rangle \cong \mathbb{Z}[X]/\langle \Phi_{2np}(X)\rangle.$$

- We have the bijection:

$$\mathsf{ct} = \sum_i Y^i \mathsf{ct}_i,$$

ct in X, Y, W-ring and ct_i in X, W-ring.

Implementation Results

```
$ pip install desilofhe
```

```
from desilofhe import GLEngine

engine = GLEngine()

secret_key = engine.create_secret_key()
matrix_multiplication_key =
engine.create_matrix_multiplication_key(secret_key)
ciphertext1 = engine.encrypt(np.ones(engine.shape), secret_key)
ciphertext2 = engine.encrypt(np.ones(engine.shape), secret_key)

multiplied = engine.matrix_multiply(
    ciphertext1, ciphertext2, matrix_multiplication_key
)
```

Implementation Results: Comparison of Operations

Table 1: Runtime for 16 batched 128×128 matrices with three levels ($n = 256, p = 17$) on an Intel i9-11900K (single-threaded)

Operations		Runtime (s)			
Category	Operation	Total	Key switching	Others	Mat. Mult. in $\mathbb{Z}_q$
Matrix Mult.	Ct-Ct matrix mult.	11.582	3.343	8.240	7.407
	(with FLINT)	**(7.237)**	(3.317)	(3.920)	(3.042)
	Pt-Ct Matrix Mult.	4.220	-	4.220	3.705
	(with FLINT)	(2.064)	-	(2.064)	(1.524)
Add/Mult.	Addition	0.018	-	0.018	-
	Hadamard Mult.	**2.779**	1.169	1.610	-
Transposition	Conjugation	1.216	1.178	0.038	-
	Transpose	1.712	1.674	0.038	-
	Conj. Trans.	1.758	1.682	0.039	-
Rotation	Row Rot.	1.059	1.082	0.023	-
	Column Rot.	0.005	-	0.005	-
	Inter-Matrix Rot.	1.115	1.093	0.022	-

Implementation Results: Comparison of Key Switching Cost

Table 2: Comparison with OpenFHE baseline (power-of-two ring)

	runtime (s)	slot count	amortized (μs)
Key switching single ct for R_q	0.004697	256×16	1.147
KS for Hadamard mult.	1.169	$256 \times 256 \times 16$	1.115
KS for ct-ct mult.	3.317	$256 \times 256 \times 16$	3.163
OpenFHE rotate (same coeff count)	0.003367	4096	0.822

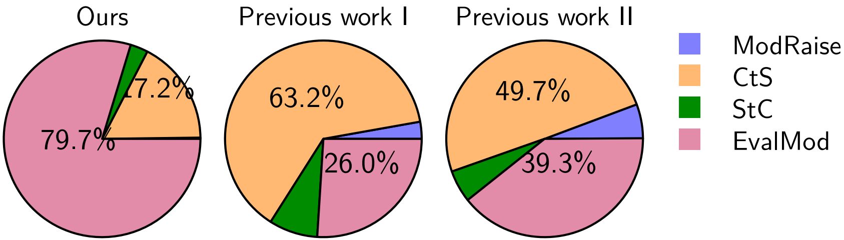
Implementation Results: GPU Acceleration

Table 3: Runtime for various parameter sets (ms): CPU (w/ FLINT) vs NVIDIA RTX 4090

n	p	level	Env.	Ct-Ct Matrix Mult		Pt-Ct Matrix Mult	Hadamard
				Mat Mult.	Total		
2^4	257	3	CPU	30.9 (34.2)	247 (245)	52 (50)	144
2^4	257	3	CUDA	0.27	35.9	-	-
2^8	17	3	CPU	8,625 (3,295)	11,989 (6,661)	4,779 (2,080)	2,406
2^8	17	3	CUDA	24.8	496	-	-
2^{10}	5	3	CPU	160,132 (46,737)	176,286 (63,066)	81,943 (24,961)	11,413
2^{10}	5	3	CUDA	451	2,822	-	-
2^5	257	9	CPU	738 (485)	4,440 (4,191)	796 (654)	2,285
2^5	257	9	CUDA	2.66	263	-	-
2^9	17	9	CPU	182,062 (60,832)	245,644 (124,210)	96,840 (35,778)	38,121
2^6	257	18	CPU	12,642 (6,868)	51,129 (45,341)	10,278 (7,241)	23,296

Application: Bootstrapping

- Our matrix multiplication can be generalized to non-power-of-two matrix dimensions.
- Native matrix multiplication enables CtS/StC with $O(1)$ key switching.
- This reduces their cost from $55–70\%$ to 20% of the total runtime.



Concluding Remarks

Engineer's Perspective:

- A new FHE scheme for matrix multiplication is here.
- Let's build privacy-preserving LLM with it.

Cryptographer's Perspective:

- There is a richer set of homomorphic operations beyond $+, \times$.
- We may be able to move more structure "inside-out".

Limitation

- Due to batching, the unit size increases by $\times n$.
- NTT for Φ_p needs more engineering.

Thank you!

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