Batch, Pack, and Prove

More Efficient Verifiable Computation for CKKS

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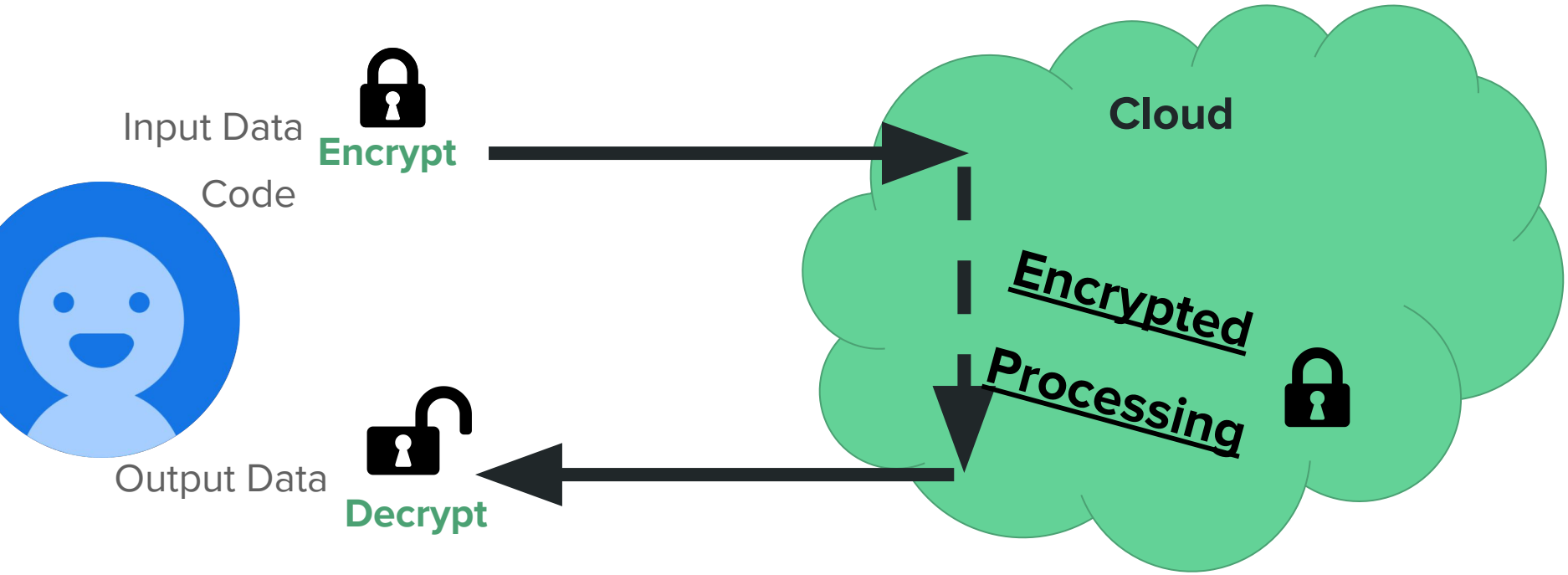
Context

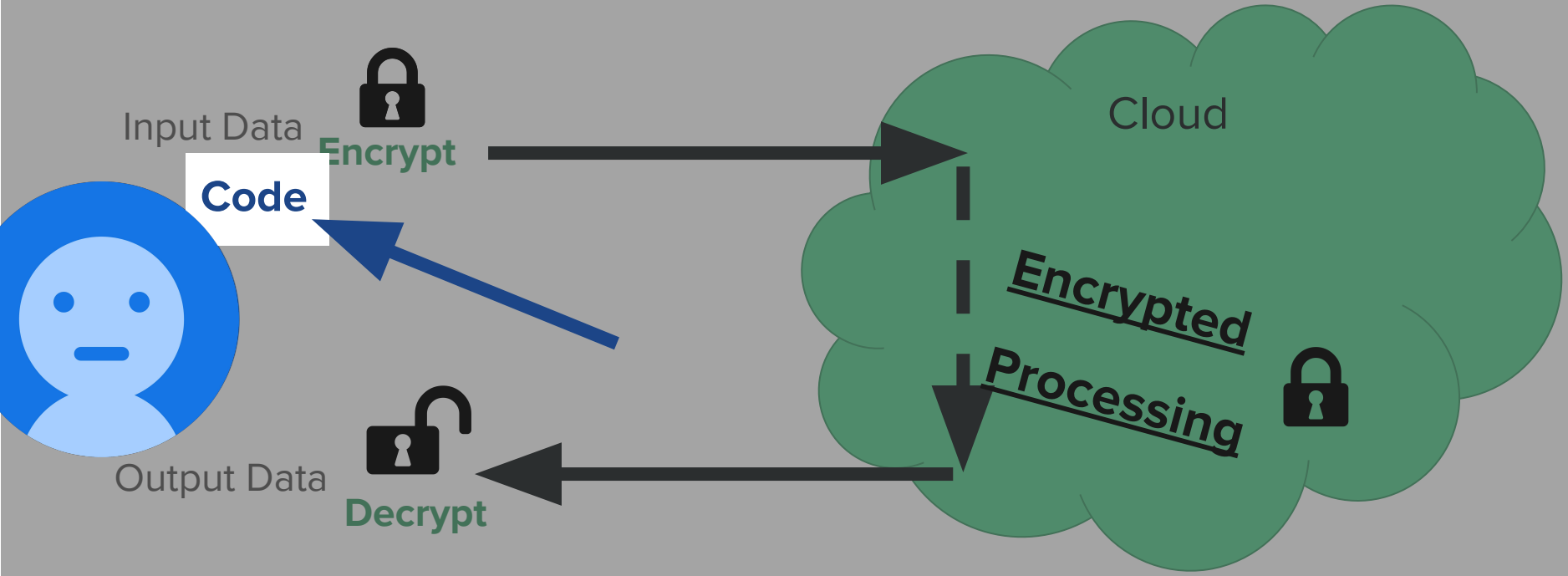
Verifiable FHE

Homomorphic Encryption (HE)



- **privacy**

Homomorphic Encryption (HE)



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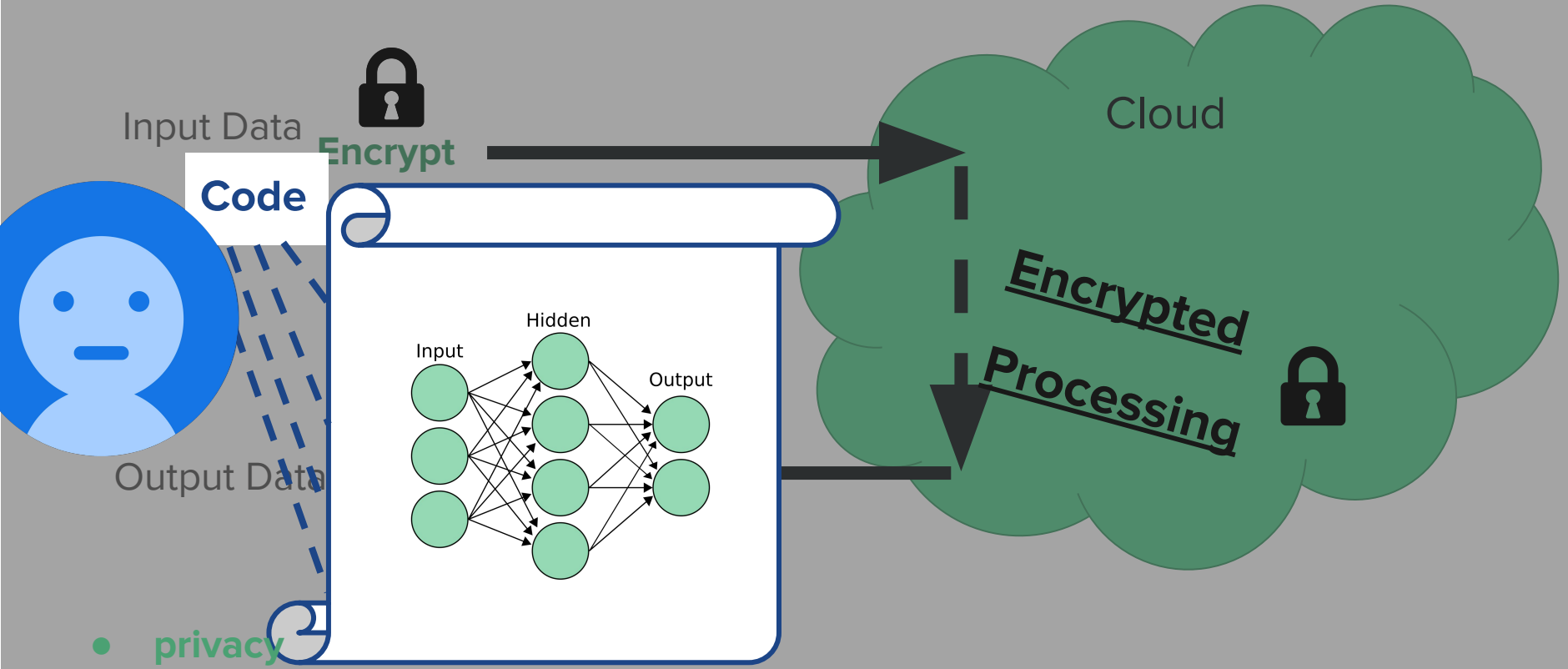

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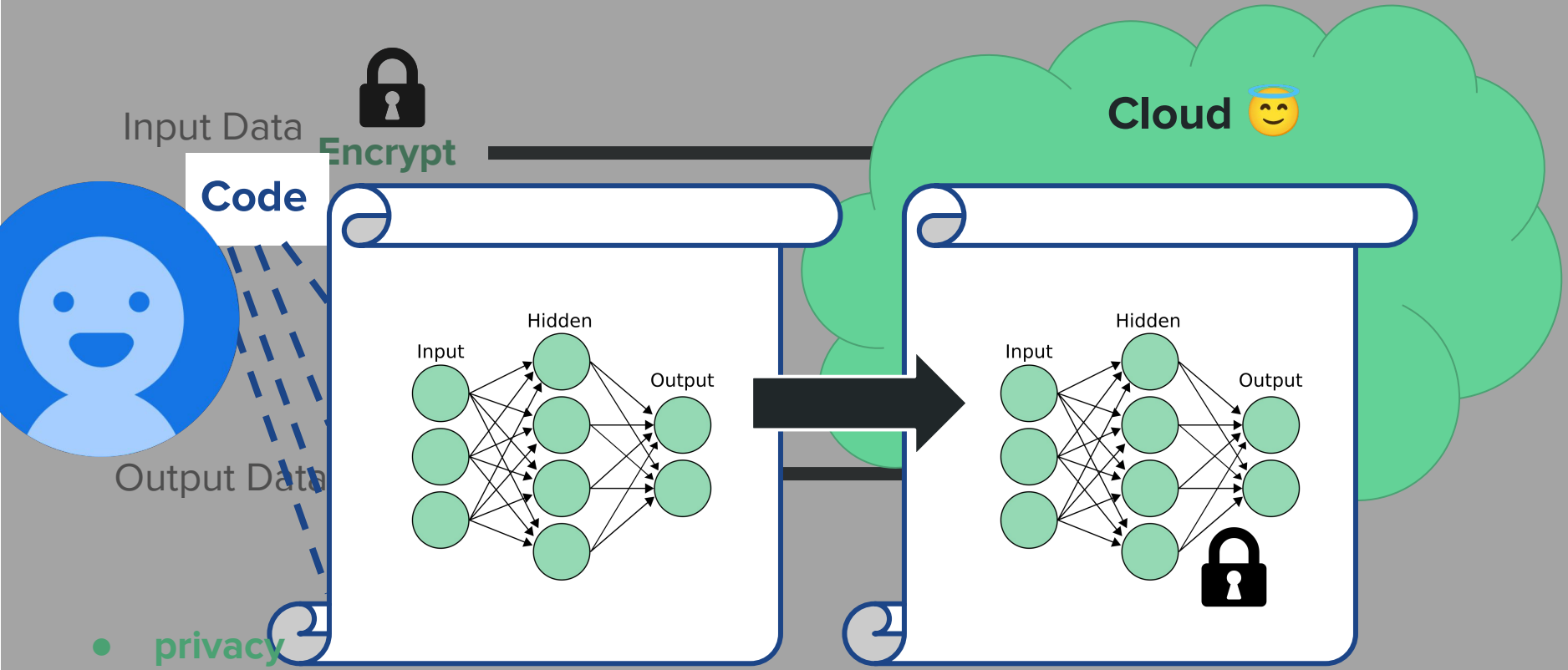

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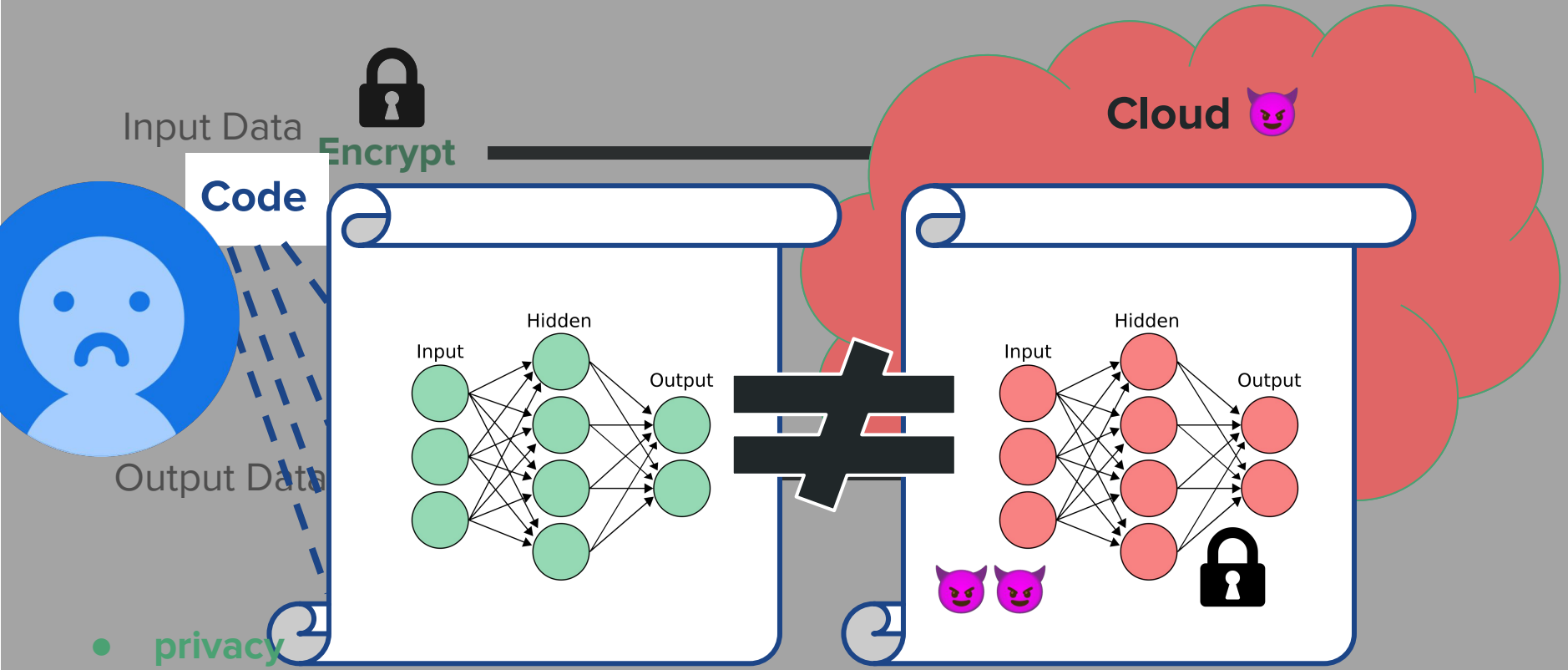


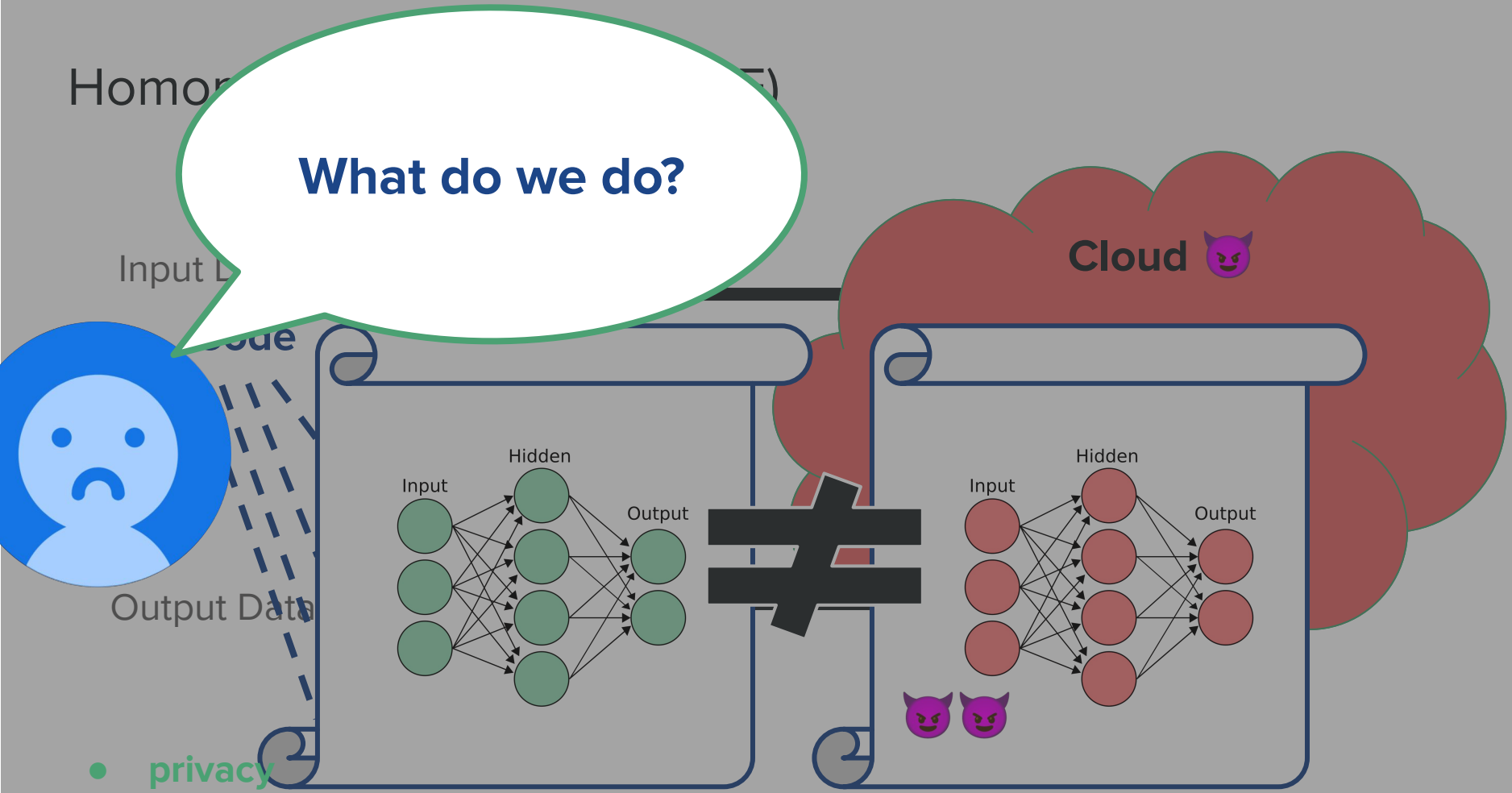
- privacy

Homor

Input D

ode

Output Data



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Homor

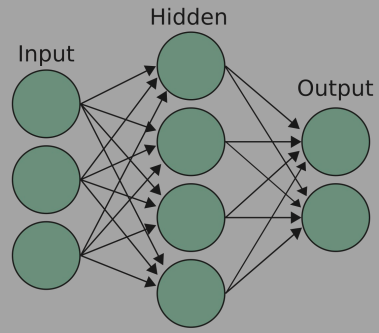
Input D

ode

Output Data

Assume a semi-honest adversary!

Cloud 😈



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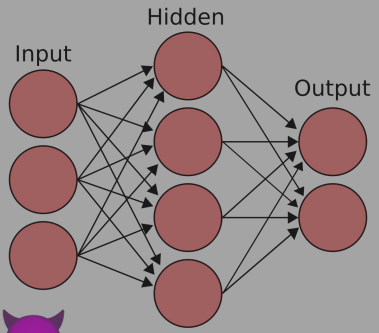


😈😈

- privacy

Homor

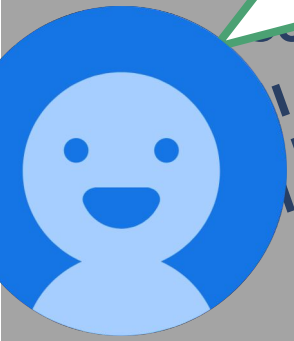
Input D

ode

Output Data

Assume a semi-honest adversary!

Everybody is happy... :)



- privacy

Homor

Assume a semi-honest adversary!

Input D

ode

Cloud 😈

Everybody is happy... :)

Input

Output

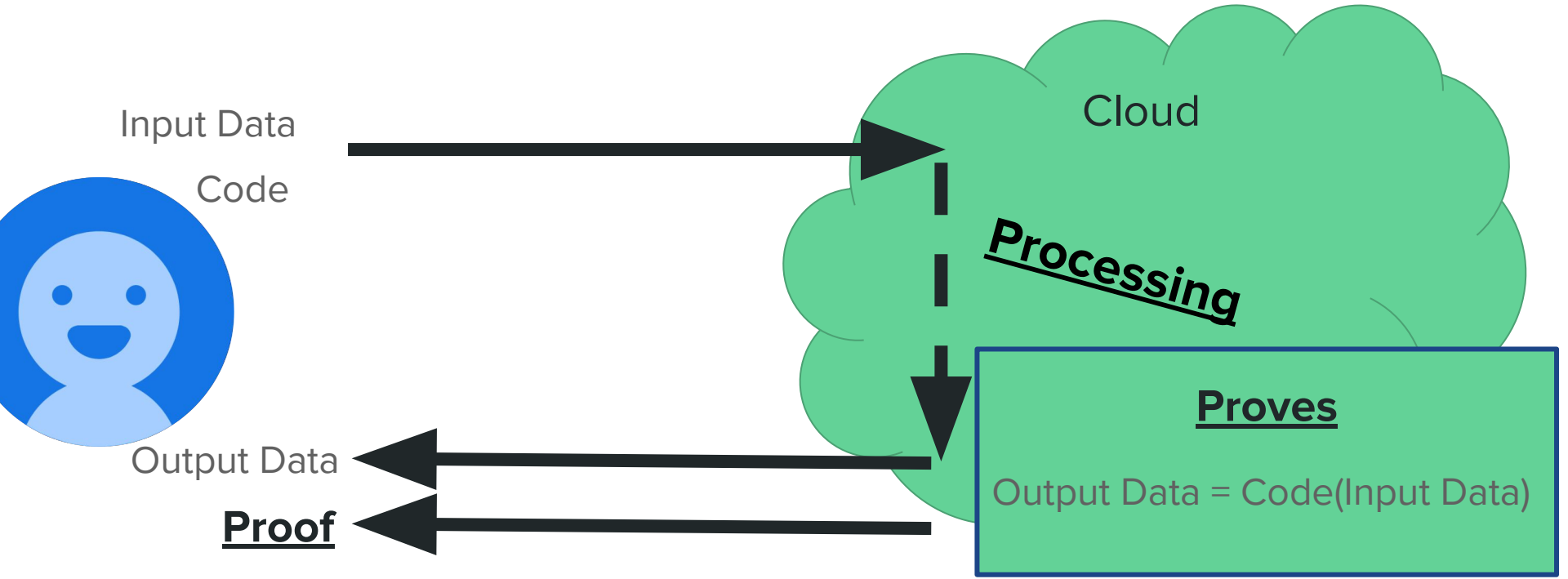
Output

Output Data

Including the malicious adversaries! :0

- privacy

Verifiable computation (VC)



- **Privacy**
- **Integrity**
- **Succinctness**

Verifia

Input D

ode

Can we have both HE and VC? Efficiently?

Cloud

Processing

Proves

Output Data = Code(Input Data)

Output Data

Proof

- **Privacy**
- **Integrity**
- **Succinctness**

VFHE and CCC^{+}25

VFHE

	Public Verification	Native R_q Arithmetic	Efficient Key Switching / Rescale	Efficient Bootstrapping	CKKS (approximate schemes)
Generic SNARK[1]	✓	✗	✗	✗	✓
Rinocchio[2]	✗	✓	✗	✗	✓
HE-IOPs[3]	!	✓	✓	✓	✗
CCC+25[4]	✓	✓	✓	?	✓

[1] A. Viand, C. Knabenhans, and A. Hithnawi, “Verifiable Fully Homomorphic Encryption,” Feb. 11, 2023, arXiv: arXiv:2301.07041
[2] C. Ganesh, A. Nitulescu, and E. Soria-Vazquez, “Rinocchio: SNARKs for Ring Arithmetic,” Journal of Cryptology, 2023
[3] D. F. Aranha, A. Costache, A. Guimarães, and E. Soria-Vazquez, “HELIOPOLIS: Verifiable Computation over Homomorphically Encrypted Data from Interactive Oracle Proofs is Practical,” in Advances in Cryptology – ASIACRYPT 2024
[4] I. Cascudo, A. Costache, D. Cozzo, D. Fiore, A. Guimarães, and E. Soria-Vazquez, “Verifiable Computation for Approximate Homomorphic Encryption Schemes,” in Advances in Cryptology – CRYPTO 2025

CCC+25 - VFHE blueprint

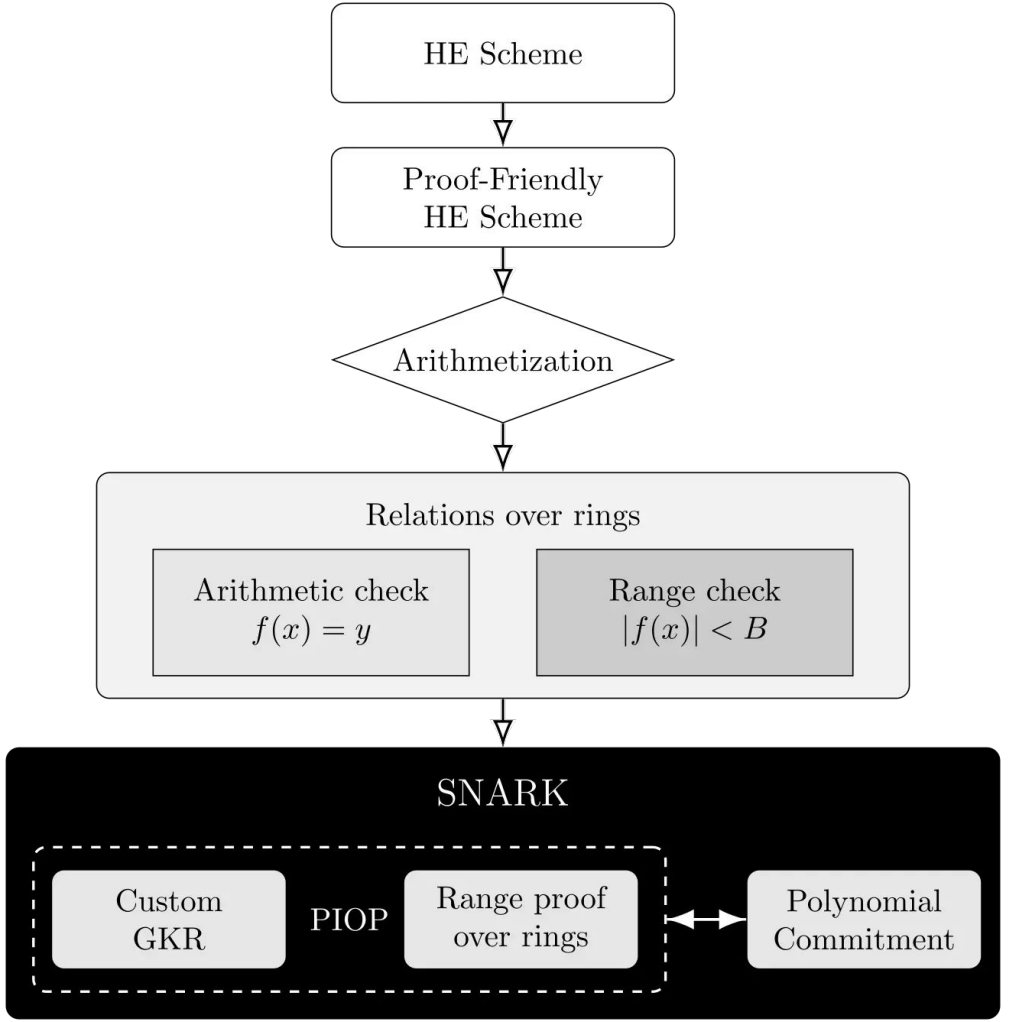
What we **did:**

- A blueprint for efficient VFHE and its instantiation to CKKS

Learn more:

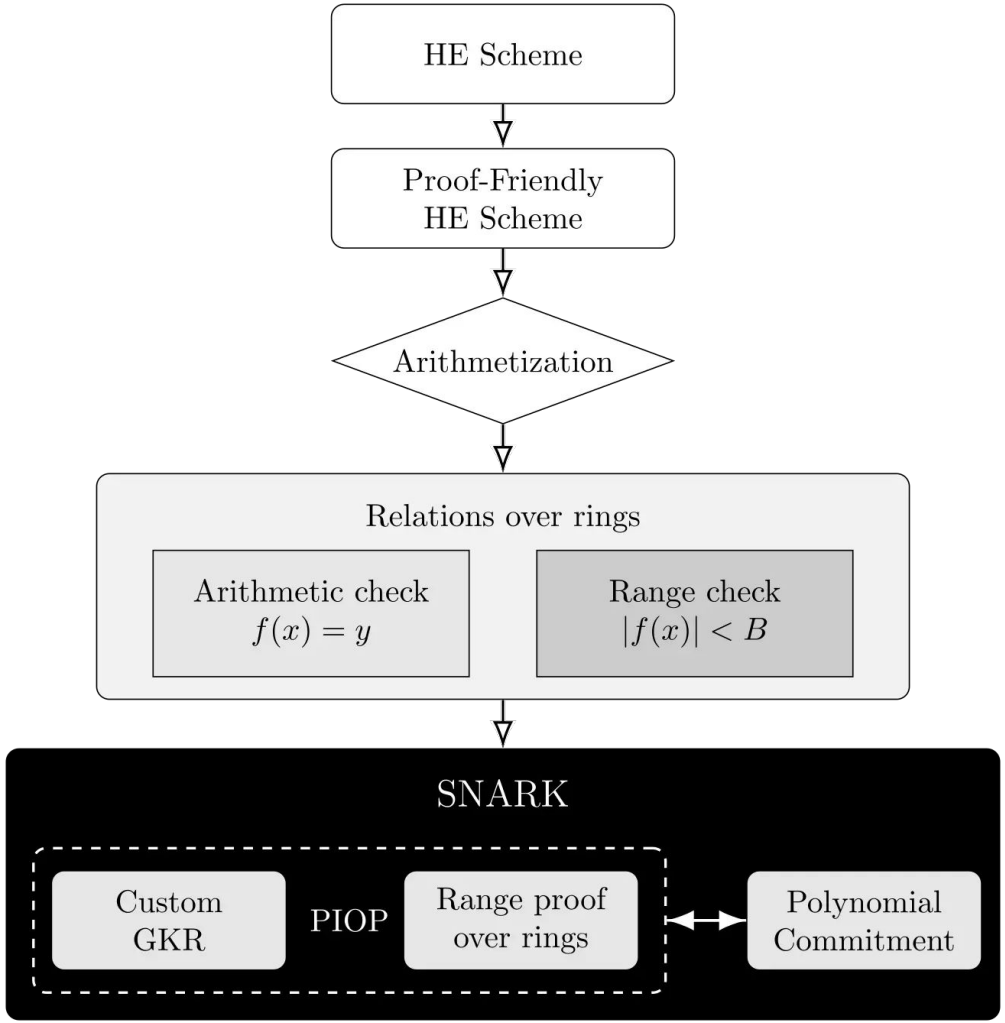
CKKS.org
blogpost:

FHE.org
2025:



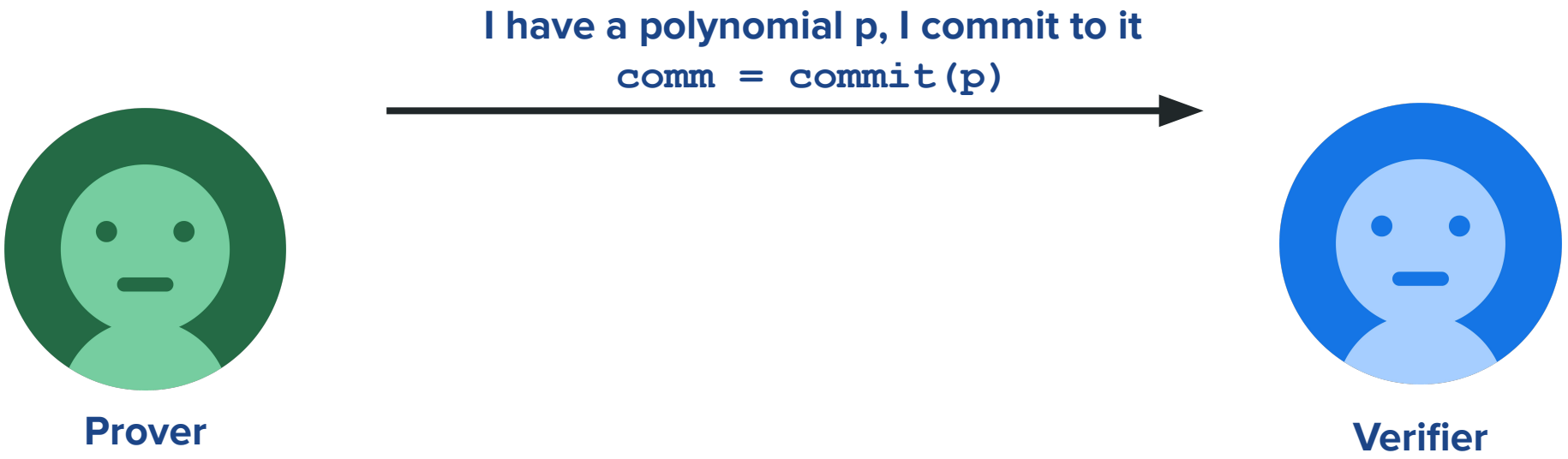
CCC+25 - VFHE blueprint

But what **didn't** we do**?**



Preliminaries: Polynomial Commitments

Polynomial commitments



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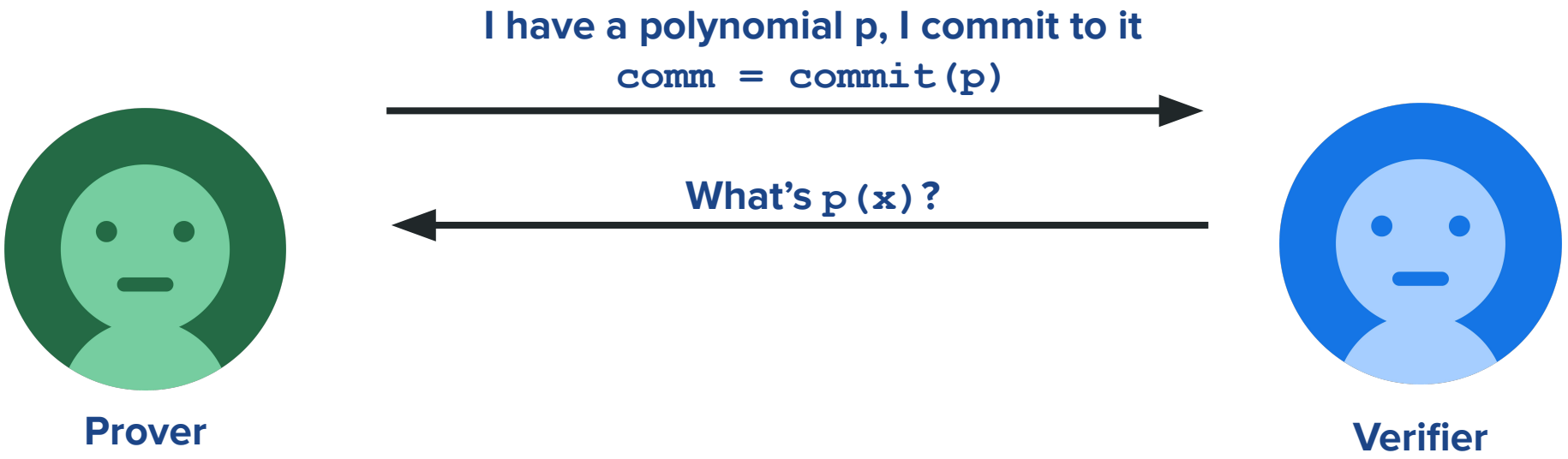

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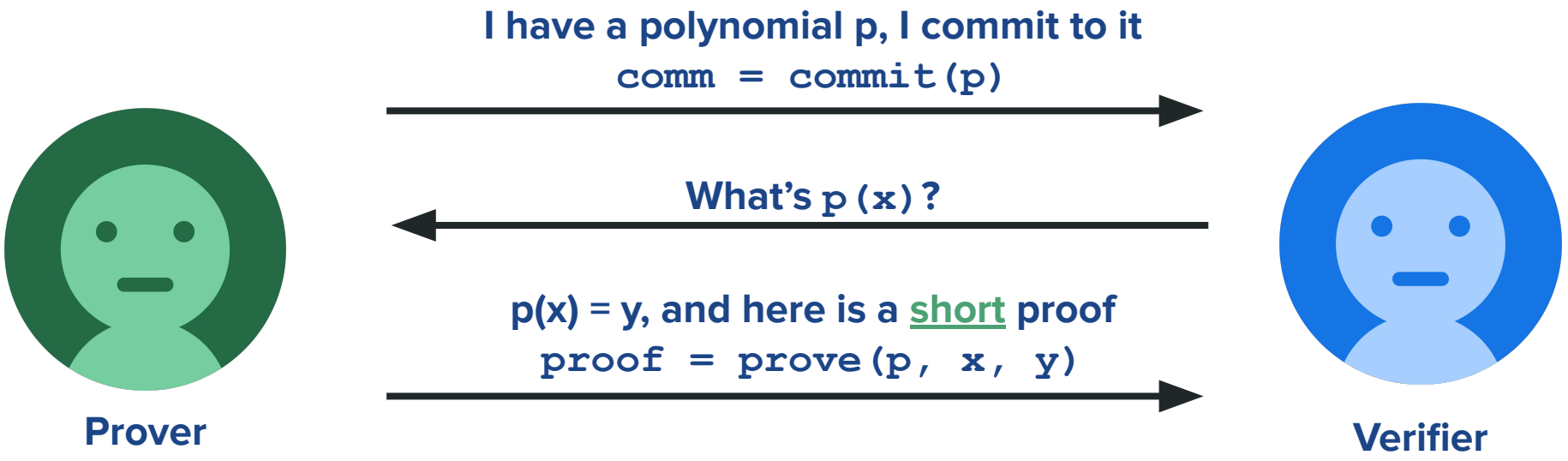

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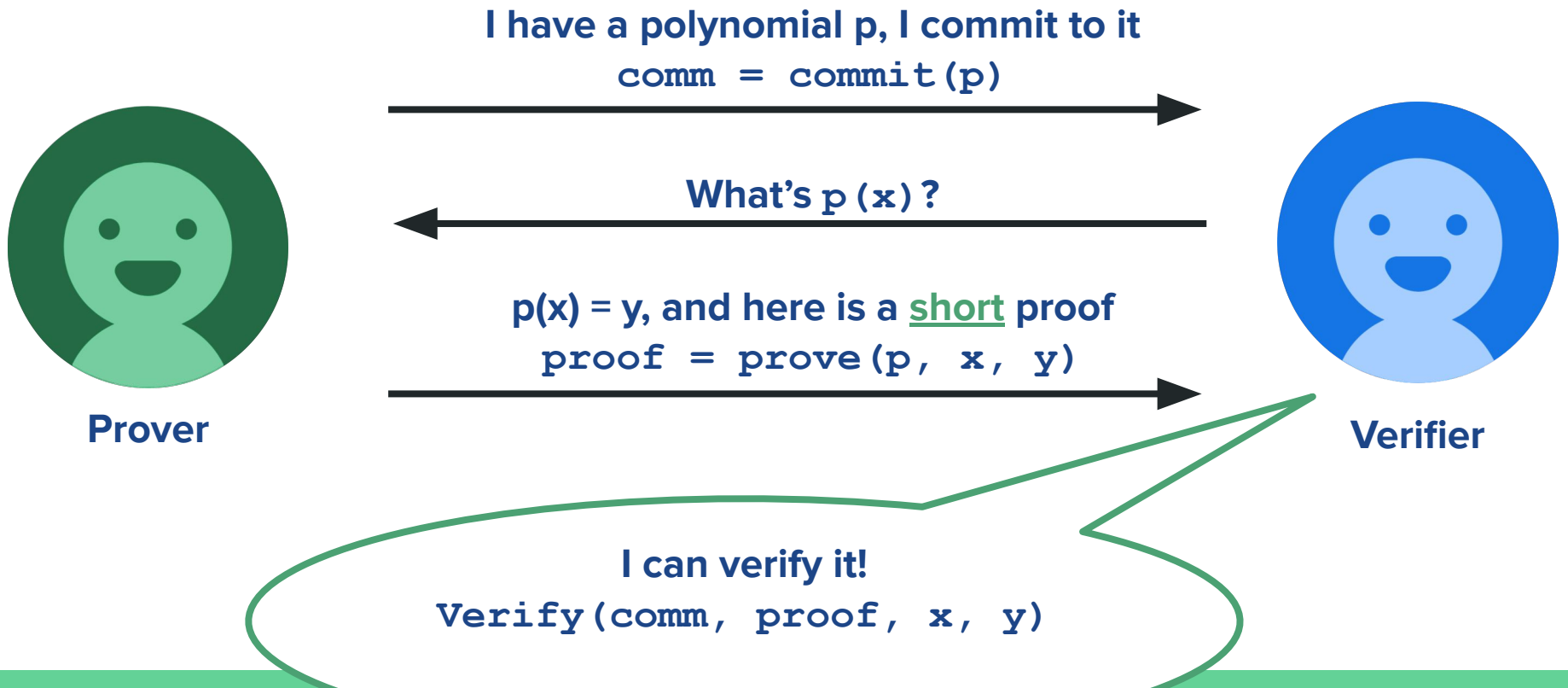


Polynomial co

Important!

x, y, and p can be anything.

For example, in a PC for R_q:

$$p \in \mathcal{R}_q[X]$$

$$x, y \in \mathcal{R}_q$$

$$y = p(x)$$

Prover

Verifier

verify(comm, proof, x, y)

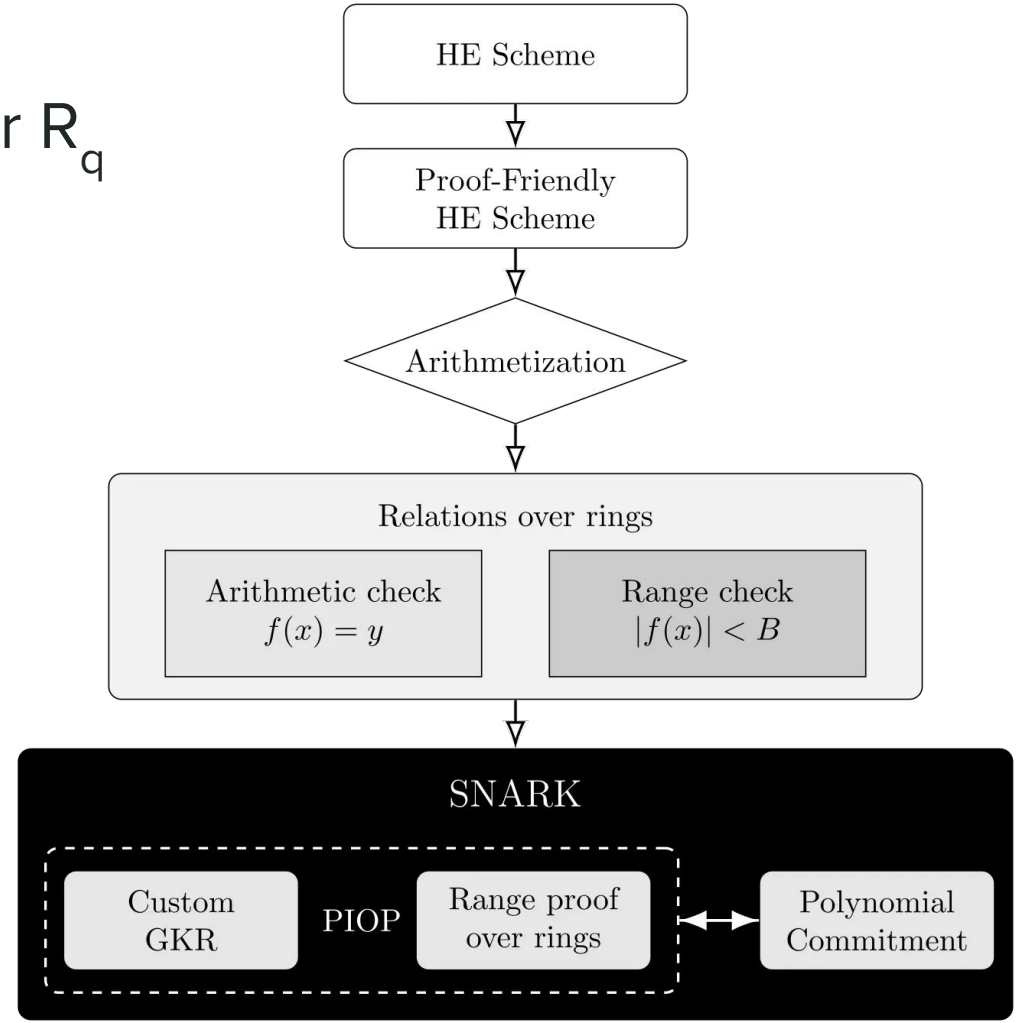
This work

This work

1. Oracle Packing
2. BatchFold
3. SparsePack

CCC+25 - A proof system for R_q

- Custom GKR for R_q
- Range checks in R_q
- Faster Brakedown PC over R_q

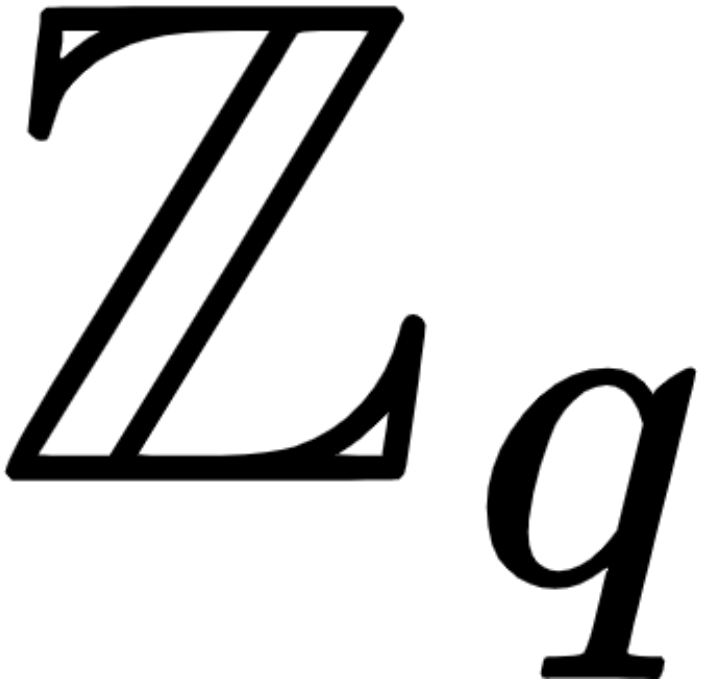


Not everything is about R_q

Not everything is about R_q

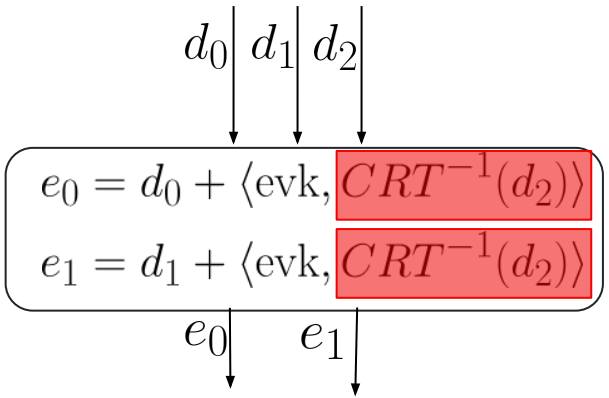
FHE also needs Z_q:

1. We scale by integers
2. We perform automorphisms (coefficient permutations)
3. We use **infinity norm** for bounding noise



CCC+25 - Range proofs for key switching

Key Switching



CCC+25 verification

1. Ask the prover to provide $\mathbf{w} = \mathbf{CRT^{-1}(d_2)}$
2. Check:
   a. $d_2 - CRT(\mathbf{w}_{\text{ks}}) \overset{?}{=} 0$
   b. $||\mathbf{w}_{\text{ks}}[i]|| < 2^k$
3. Compute and check:

$$e_0 = d_0 + \langle \text{evk}, w_{\text{ks}} \rangle$$
$$e_1 = d_1 + \langle \text{evk}, w_{\text{ks}} \rangle$$

CCC+25 - Range proofs

Every key switching produces several: $||\mathbf{w}_{\text{ks}}[i]|| < 2^k$

- Creates a big vector V with all $\mathbf{w}_{\text{ks}}$ (notice that $\mathbf{w}_{\text{ks}}$ is a polynomial R_q)
- Prove that all elements in V have **coefficients** bounded by 2^k using lookup arguments

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CCC+25 - Range proofs

This proof is about $\mathbb{Z}_q$

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CCC+25 - Lasso-style lookup arguments

1. **Decomposes R_q elements into Z_B for some small B**
2. **Proves that the decomposed values are in tables on size B**

CCC+25 - Range proofs

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CCC+25 - Lasso-style lookup arguments

1. **Decomposes R_q elements into Z_B for some small B**
   - **It needs to commit to these decomposed elements**
2. **Proves that the decomposed values are in tables on size B**

CCC^{+}25 - Range proofs

1. Every key switching produces several: $||\mathbf{w}_{\mathrm{ks}}[i]|| < 2^{k}$
2. Every R_q element is decomposed in Z_B elements, which are committed to using a PC for R_q

CCC+25 - Range proofs

R_q has O(N) such elements

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CCC+25 - Range proofs

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$O(N^2)$ total costs

Can't we use a PC for Z_q instead?

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CCC+25 - Range proofs

R_q has O(N) such elements

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Can't we use a PC for Z_q instead?

Not yet! We still need R_q for the rest of the protocol

Virtual Oracles and Oracle Packing

The problem

- We have <u>many</u> small Z_B elements we need to commit
- But we only have a polynomial commitment for R_q

Solution

- <u>Pack</u> many small Z_B elements in R_q
- Create virtual oracles for Z_B
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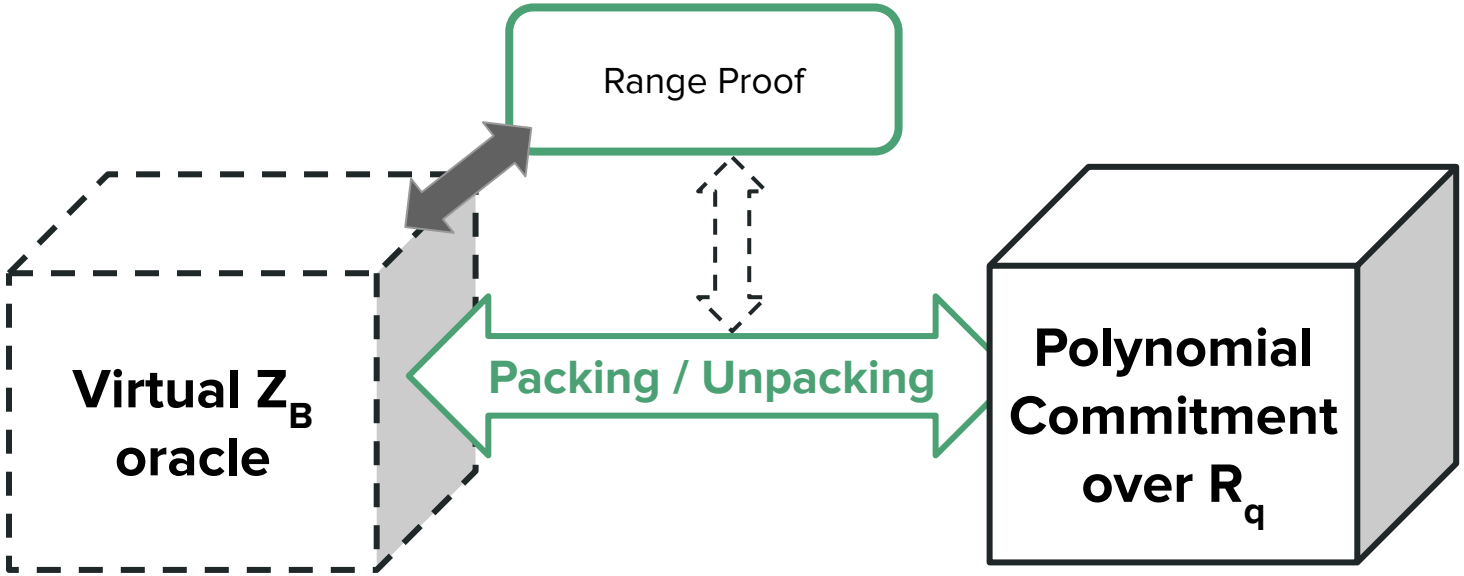

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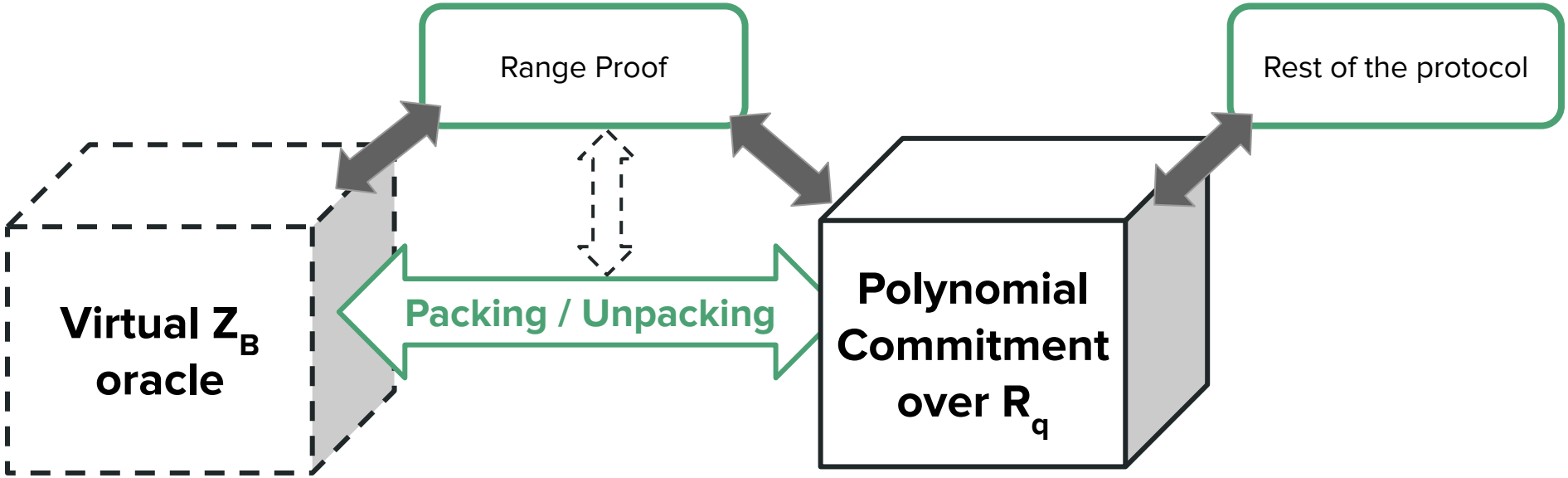

Solution

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1. PC for range proofs are O(N) times faster
2. Beyond range proofs:

<u>We can use this whenever we need commitments to Z_q</u>

Rest of the protocol

Virtual Z_B oracle

Packing / Unpacking

Polynomial Commitment over R_q

Polynomial Commitment

Polynomial Commitments

We need a (multivariate) polynomial commitment scheme for our ring

- CCC^{+}25 uses **Brakedown** [1]
  - **Linear time prover**
  - **Square-root time verifier**

[1] A. Golovnev, J. Lee, S. Setty, J. Thaler, and R. S. Wahby, “Brakedown: Linear-Time and Field-Agnostic SNARKs for R1CS,” in Advances in Cryptology – CRYPTO 2023

Polynomial Commitments

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  - **Quasilinear prover**
  - **Polylog verifier**

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Polynomial Commitments

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  - **Polylog verifier**
- Can we do better?

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Basefold → BatchFold

Oracle access to polynomial evaluations

$$p \in \mathcal{R}_q[X]$$

$$x, y \in \mathcal{R}_q$$

Prove: $y = p(x)$

Oracle access to polynomial evaluations

$$p_i \in \mathcal{R}_q[X] \qquad x_i, y_i \in \mathcal{R}_q$$

$$\texttt{Prove:}\ y_i = p_i(x_i)$$

Oracle access to polynomial evaluations

$$p_i \in \mathcal{R}_q[X] \qquad x_i, y_i \in \mathcal{R}_q$$

$$\textbf{for } i \leftarrow 0 \textbf{ to } O(\log_2 |C|) \textbf{ do:}$$

`Prove:` $y_i = p_i(x_i)$

Oracle access to polynomial evaluations

$$p_i \in \mathcal{R}_q[X] \qquad x_i, y_i \in \mathcal{R}_q$$

Let's batch them!

for $i \leftarrow$ [illegible] $\mathcal{O}(\log_2 |\mathcal{O}|)$ **do:**

Prove: $y_i = p_i(x_i)$

Original Basefold[1]

- Works over **Fields**
- For ***M*** **polynomials of size *n*:**

$$O(M \log^2 n)$$

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Our Batched Ring Basefold

- Works over our **proof-friendly CKKS Ring**
- For M polynomials of size n:

$$O(M \log n + \log^2 n)$$

Original Basefold[1]

- Works

- For *M* polynomials of size *n*:

$$O(M \log^2 n)$$

Our Batched Ring Basefold

- roof-friendly

- For M poly ials of size n:

$$O(M \log n + \underline{\log^2 n})$$

Big “constant” factor here!

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Original Basefold[1]

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Our Batched Ring Basefold

- roof-friendly

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Can we go further?

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Remember the **proof-friendly ring**

The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \quad q = \prod_{i=1}^{L} p_i \cong$$

R_{p_1}

$$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_1 \cong$$

$\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$ $\mathbb{F}_{p_1^d}$

R_{p_2}

$$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_2 \cong$$

$\mathbb{F}_{p_2^d}$ $\mathbb{F}_{p_2^d}$ $\mathbb{F}_{p_2^d}$ $\mathbb{F}_{p_2^d}$

R_{p_3}

$$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_3 \cong$$

$\mathbb{F}_{p_3^d}$ $\mathbb{F}_{p_3^d}$ $\mathbb{F}_{p_3^d}$ $\mathbb{F}_{p_3^d}$

$$d = 1 \longrightarrow \mathbb{F}_{p_1} \qquad d = 2 \longrightarrow \mathbb{F}_{p_1^2}$$

Remember the Ring

$$X^N + 1 = \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \mod p_1$$

- **Fully-splitting NTT-friendly ring:**

$$d = 1 \longrightarrow \mathbb{F}_{p_1}$$

- **Almost-Fully-splitting Almost-NTT-friendly ring:**

$$d = 2 \longrightarrow \mathbb{F}_{p_1^2}$$

The polynomial ring $R_q = \mathbb{Z}_q[X]/(X^N + 1)$

$$R_q \cong \prod_{i=1}^{L} R_{p_i} \quad \text{where } q = \prod_{i=1}^{L} p_i$$

$$R_{p_1} \cong \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \bmod p_1 \cong \mathbb{F}_{p_1^d} \times \mathbb{F}_{p_1^d} \times \mathbb{F}_{p_1^d} \times \mathbb{F}_{p_1^d}$$

$$R_{p_2} \cong \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \bmod p_2 \cong \mathbb{F}_{p_2^d} \times \mathbb{F}_{p_2^d} \times \mathbb{F}_{p_2^d} \times \mathbb{F}_{p_2^d}$$

$$R_{p_3} \cong \prod_{i=1}^{k} (X^d - \zeta^{2i-1}) \bmod p_3 \cong \mathbb{F}_{p_3^d} \times \mathbb{F}_{p_3^d} \times \mathbb{F}_{p_3^d} \times \mathbb{F}_{p_3^d}$$

- Efficient arithmetic with incomplete NTTs!
- Soundness
  - Proportional to $|p_i^d|$

1. **These fields are big enough for soundness**
2. **For each prime p_i, they are isomorphic to each other.**

$$q[X]/(X^N+1)$$

$$X^N+1 \equiv \prod_{i=1}^{k}(X^d-\zeta^{2i-1}) \mod p_1 \cong \mathbb{F}_{p_1^d}\ \mathbb{F}_{p_1^d}\ \mathbb{F}_{p_1^d}\ \mathbb{F}_{p_1^d}$$

$$X^N+1 \equiv \prod_{i=1}^{k}(X^d-\zeta^{2i-1}) \mod p_2 \cong \mathbb{F}_{p_2^d}\ \mathbb{F}_{p_2^d}\ \mathbb{F}_{p_2^d}\ \mathbb{F}_{p_2^d}$$

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q

R_{p_3}

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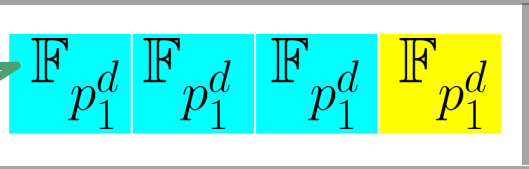
$$q[X]/(X^N+1)$$

$$\mod p_1$$

$$\mod p_2$$



Let's batch them!

- Efficient arithn
- Soundness
  - Proportional to $|p_i^d|$

Original Basefold[1]

- Works over **Fields** (adapted to rings as in CCC+25)
- For *M* polynomials of size *n*:

$$O(NLM \log^2 n)$$

[1] H. Zeilberger, B. Chen, and B. Fisch, "BaseFold: Efficient Field-Agnostic Polynomial Commitment Schemes from Foldable Codes," in Advances in Cryptology – CRYPTO 2024

BatchFold

- Works over $\mathbb{F}_{p^d}$
- Supports our **proof-friendly CKKS Ring** (by isomorphism)
- For M polynomials of size n:

$$O(NLM \log n + L \log^2 n)$$

N = Ring Degree

L = Number of RNS components

Origina

BatchFold

Big “constant” factor here!

- Works
  rings as in CCC+25)
- For *M* polynomials of size *n*:

$$O(NLM \log^2 n)$$

- ur proof-friendly
  CKKS
  y isomorphism)
- For M poly ls of size n:

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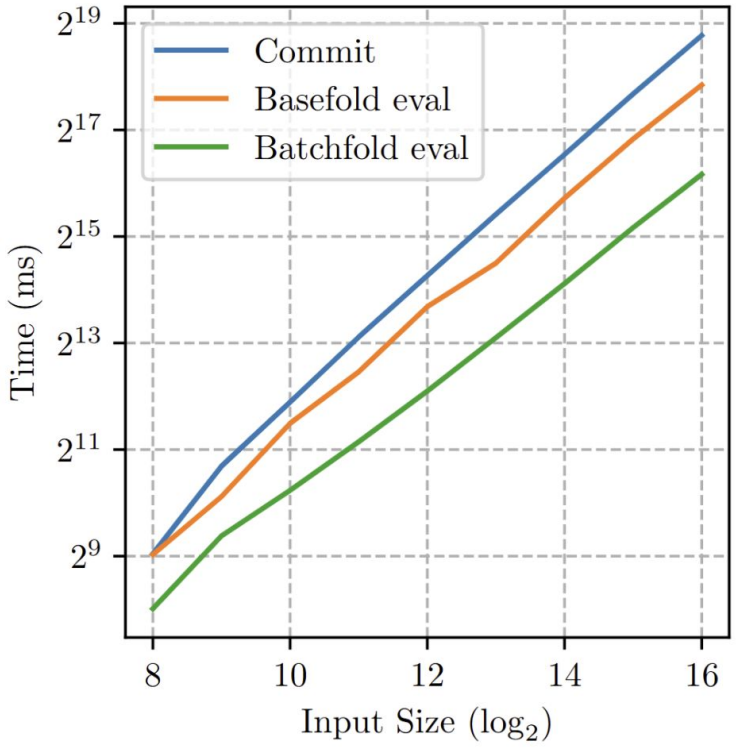
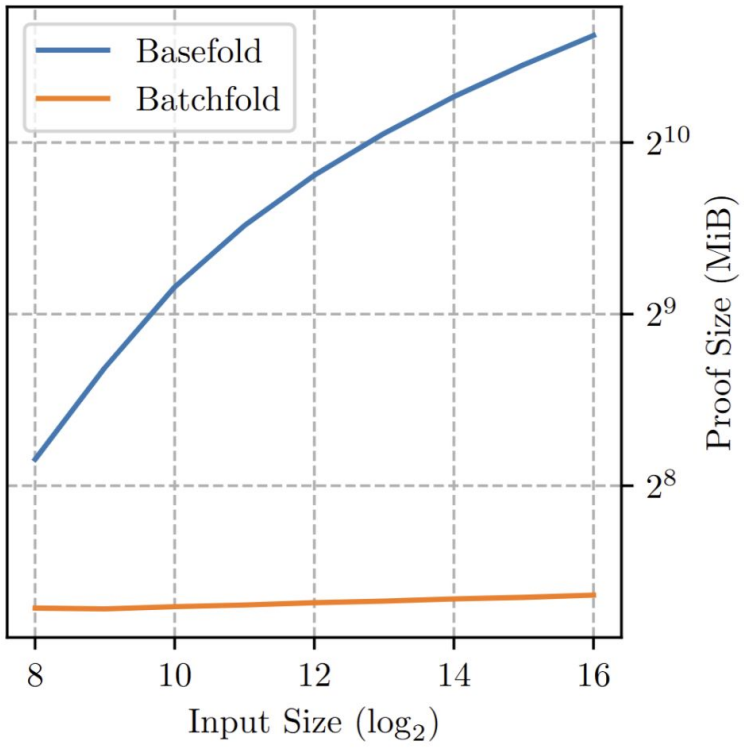
BatchFold



(a) Execution time

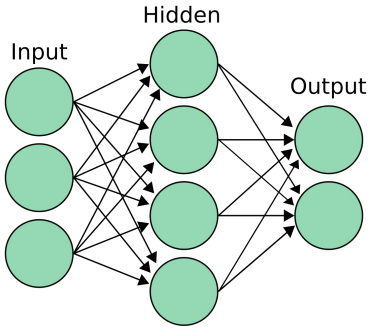


(b) Proof size

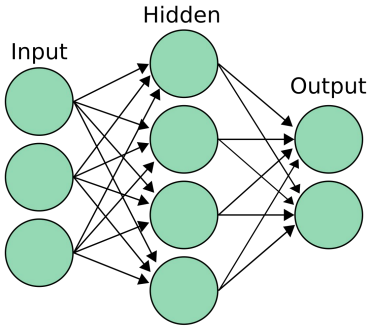
VFHE with GKR in practice

How do we prove a circuit?

```
Prove( HE(                    ) )
```



How do we prove a circuit?



NN Circuit
(using Keras)

Pure python
code
(* , + , -)

CKKS

HE evaluation
using CKKS

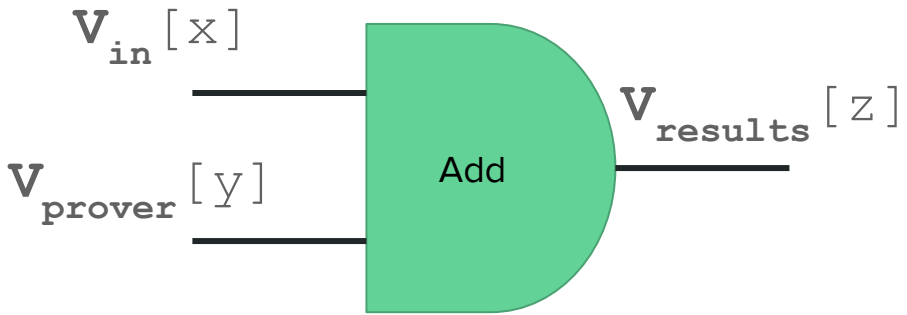
GKR

Custom GKR
circuit
(to prove HE
evaluation)

How do we **verify** this circuit?

GKR circuit example:

- Wires
  - V_{in}
  - V_{prover}
  - $V_{results}$
- Gates
  - Add



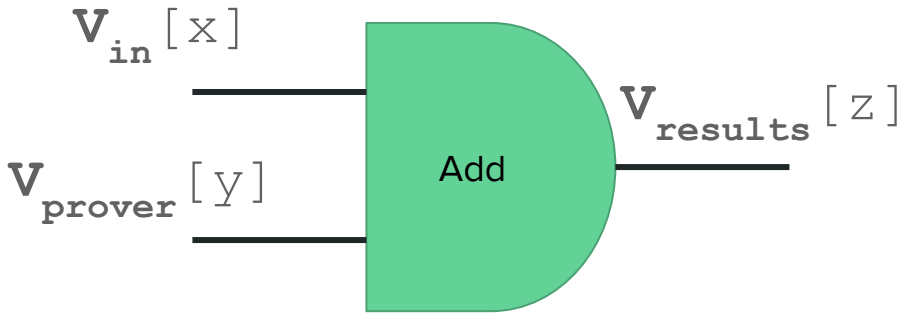
How do we **verify** this circuit?

Representing it as polynomials

Wires

1. Create a polynomial
   $p_{Vin} = 0$

2. **for** x = 0 **to** size(V_{in}):
   $p_{Vin}(x) = V_{in}[x]$



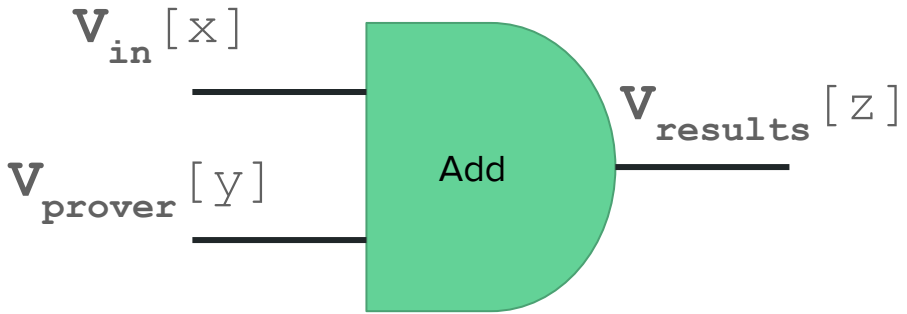
How do we **verify** this circuit?

Representing it as polynomials

Gates

1. Create a polynomial
   $p_{Add} = 0$

2. **for all** x,y,z:
   If Add(x,y,z) exists:
   $p_{add}(x|y|z) = 1$
   else:
   $p_{add}(x|y|z) = 0$



The verifier needs oracle access to these polynomials

Wires

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   else:
   $p_{add}(x|y|z) = 0$

The verifier needs oracle access to these polynomials

We can use a polynomial commitment for this one

Wires

1. Create a p[illegible]al
   $p_{Vin} = 0$

2. **for** x <= 0 **to** size(V_{in}):
   $p_{Vin}(x) = V_{in}[x]$

1. Create a polynomial
   $p_{Add} = 0$

2. **for all** x,y,z:
   If Add(x,y,z) exists:
   $p_{add}(x|y|z) = 1$
   else:
   $p_{add}(x|y|z) = 0$

The verifier needs oracle access to these polynomials

But this one is too big!

Wires

1. Create a polynomial
   $p_{Vin} = 0$

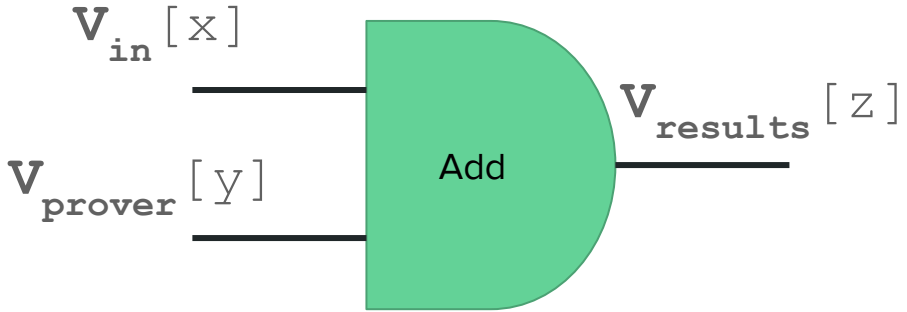
2. **for** x <- 0 **to** size(V_{in}):
   $p_{Vin}(x) = V_{in}[x]$

1. Create a polynomial
   $p = 0$

2. **for** all x,y,z:
   If Add(x,y,z) exists:
   $p_{add}(x|y|z) = 1$
   else:
   $p_{add}(x|y|z) = 0$

Solutions

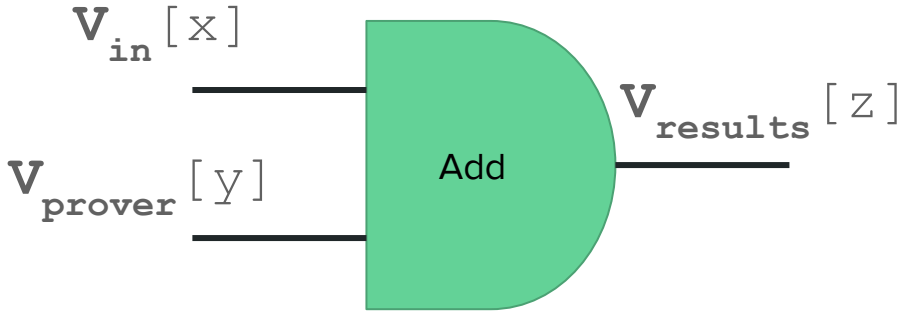
1. Log-space description
2. Sparse polynomial commitments



Solutions

1. Log-space description
2. Sparse polynomial commitments

We do both!



Improved Arithmetization

Gates

$$\texttt{rescon}_1(\mathbf{z},\mathbf{x}) = \begin{cases} 1, & \text{if wire } \mathbf{x} \text{ holds the } \mathbf{z}\text{-th } c' \text{ value.} \\ 0, & \text{otherwise.} \end{cases}$$

$$\texttt{rescon}_3(\mathbf{z},\mathbf{x}) = \begin{cases} -1, & \text{if wire } \mathbf{x} \text{ holds the } \mathbf{z}\text{-th remainder.} \\ -\mathsf{p}_l, & \text{if } \mathbf{x} \text{ belongs to the } l\text{-th HE layer and holds the } \mathbf{z}\text{-th quotient.} \\ 0, & \text{otherwise.} \end{cases}$$

$$\texttt{bdcon}_2(\mathbf{z},\mathbf{x}) = \begin{cases} 1, & \text{if wire } \mathbf{x} \text{ holds the } \mathbf{z}\text{-th } d_2 \text{ value.} \\ 0, & \text{otherwise.} \end{cases}$$

$$\texttt{bdcon}_3(\mathbf{z},\mathbf{x}) = \begin{cases} -\mathsf{PW}_{\omega_l}(1)[i], & \text{if } \mathbf{x} \text{ belongs to the } l\text{-th HE layer and holds} \\ & \text{the } i\text{-th decomposition of the } \mathbf{z}\text{-th } d_2 \text{ value.} \\ 0, & \text{otherwise.} \end{cases}$$

$$\texttt{evk}(\mathbf{z},\mathbf{x}) = \begin{cases} \mathsf{evk}_{l,b}[i], & \text{if wire } \mathbf{x} \text{ belongs to the } l\text{-th HE layer and} \\ & \text{holds the } i\text{-th decomposition of the } d_2 \text{ value} \\ & \text{corresponding to the } \mathbf{z}\text{-th } c'[b], \text{ where } b \in \{0,1\}. \\ 0, & \text{otherwise.} \end{cases}$$

$$\texttt{Xmult}(\mathbf{z},\mathbf{x},\mathbf{y}) = \begin{cases} 1, & \text{if } V(\mathbf{x}) \cdot V(\mathbf{y}) \text{ is added to } V_2(\mathbf{z}). \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

Prescriptive wires

	LSPs		Middle positions		MSPs	
	Bits	Meaning	Bits	Meaning	Bits	Meaning
$\tilde{V}_0$	$\mathtt{T}_0, \mathtt{T}_1$	Gate type	$\mathtt{Z}$	Gate number (for that type)	-	-
$\tilde{V}_1$	$\mathtt{Z}_{[\delta]}$	HE level	$\mathtt{Z}_\delta$	key-switch output type: c'_0 vs c'_1	$\mathtt{Z}_{(\delta,\mu)}$	wire on level
$\tilde{V}_{2,d_{01}}$	$\mathtt{Z}_{[\delta]}$	HE level	$\mathtt{Z}_\delta$	pre-mult output type: d_0 vs d_1	$\mathtt{Z}_{(\delta,\mu)}$	wire on level
$\tilde{V}_{2,d_2}$	$\mathtt{Z}_{[\delta]}$	HE level	$\mathtt{Z}_{[\delta,\mu-1)}$	wire carrying pre-mult output d_2	-	-
$\tilde{V}_2$	$\mathtt{Z}_0$	d_{01} vs d_2	As $\tilde{V}_{2,d_{01}}$ or $\tilde{V}_{2,d_2}$			
$\tilde{V}_{3,BD}$	$\mathtt{Z}_{[\delta]}$	HE level	$\mathtt{Z}_{[\delta,\mu-1)}$	wire carrying pre-mult output d_2	$\mathtt{X}_{[\delta]}$	specific p_i
$\tilde{V}_{3,quot}$	$\mathtt{Y}_{[\delta]}$	HE level	$\mathtt{Y}_\delta$	mult output type: c''_0 vs c''_1	$\mathtt{Y}_{(\delta,\mu)}$	wire on level
$\tilde{V}_{3,rc_3}$	$\mathtt{Z}_{[\delta]}$	HE level	$\mathtt{Z}_{[\delta,\mu)}$	$\mathtt{Z}_\delta$: c''_0 vs c''_1 | $\mathtt{Z}_{(\delta,\mu)}$: wire on level	$\mathtt{X}_0$	rmd vs quot

Table 1: Variable information for the different intermediate-values MLEs. LSPs = Least significant positions, MSPs = Most significant positions.

SparsePack

Problem: Existing **Sparse** Polynomial Commitments (e.g. Spark) are not efficient enough for R_q

Solution:

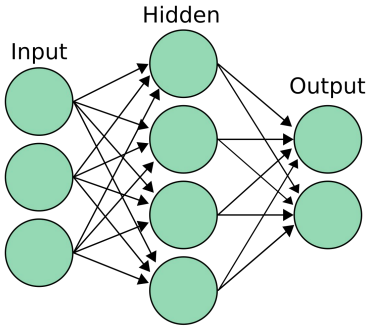
Oracle Packing + **BatchFold** → **SparsePack**

Results

Applications

We consider 3 small Neural Network circuits

- Small NNs for binary classification:
  - 30 inputs → 2 outputs
  - 2 hidden layers each
- Activation: $f(x) = x^2$



Model	Ctxt Multiplications	Circuit size	Dense circuit size
(128, 64)	192	2^{37}	2^{15}
(64, 16)	80	2^{33}	2^{13}
(32, 16)	28	2^{30}	2^{12}

Spark vs SparsePack

Table 2: Performance of our SparsePack and comparison with Spark. Notice that our solution does not require execution-time commitments.

Model (n_1, n_2)	Spark					SparsePack			
	`SumCheck`		`Comm.`	`Eval`		`SumCheck`		`Eval`	
	Size	Time	Time	Size	Time	Size	Time	Size	Time
$(128, 64)$	867.9 MiB	5m54s	20m57s	3.8 GiB	14m42s	54.3 MiB	25m58s	162.7 MiB	1m
$(64, 16)$	666.0 MiB	1m31s	20m57s	3.8 GiB	3m31s	48.8 MiB	6m40s	159.5 MiB	22s
$(32, 16)$	575.1 MiB	45s	20m57s	3.7 GiB	1m45s	36.1 MiB	1m20s	159.5 MiB	7s

Range proofs

Single-threaded execution:

Table 3: Timing performance of our range proofs

Model (n_1, n_2)	SumCheck	Com	Eval	Total
$(128, 64)$	12s	9m27s	5m8s	14m47s
$(64, 16)$	7s	4m10s	2m33s	6m49s
$(32, 16)$	6s	1m59s	1m27s	3m32s

CCC+25: 25s per multiplication in **48** threads

Thank you!

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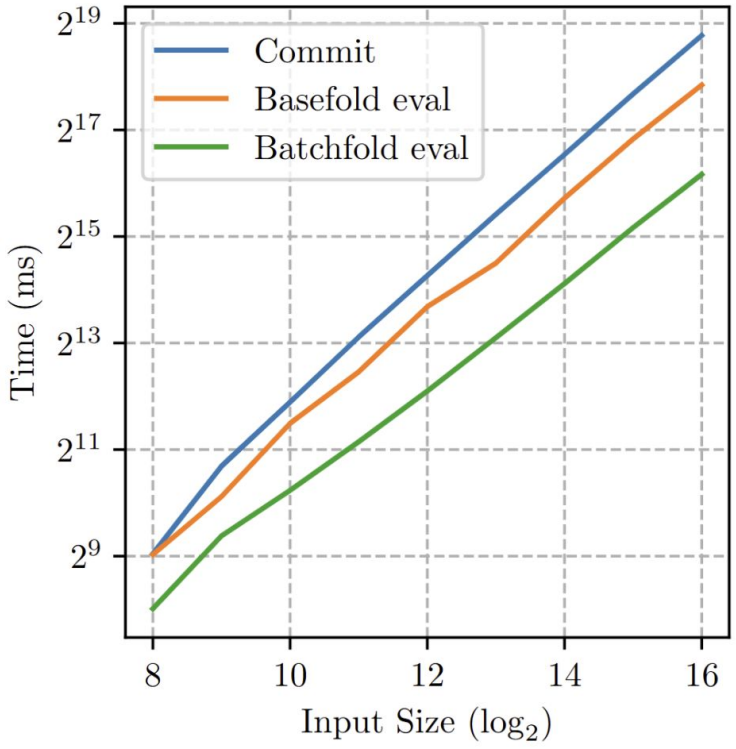
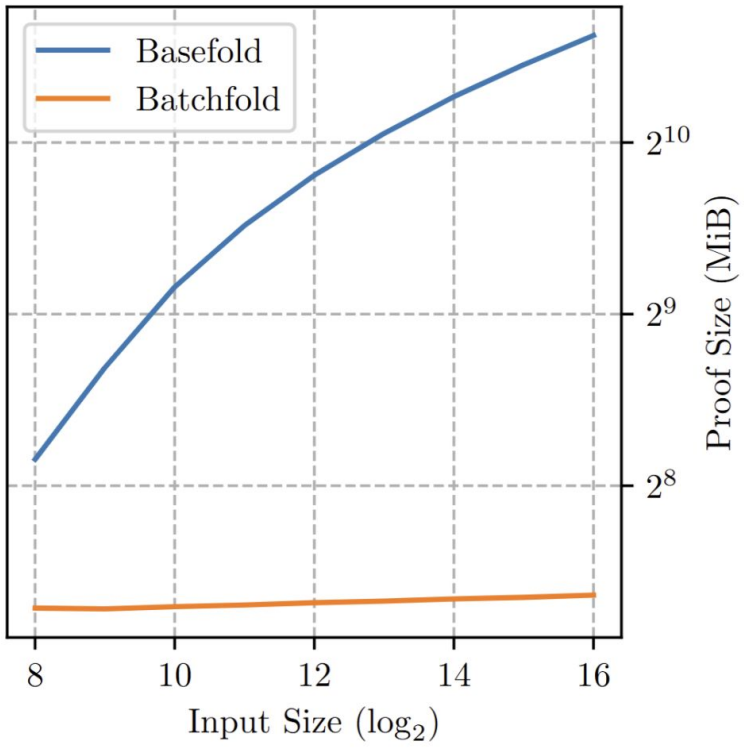
BatchFold



(a) Execution time



(b) Proof size

Images used in this presentation

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