



# To Search and Protect:

## An MPC/FHE Approach to Encrypted Search

Shai Halevi - AWS

# Theme: FHE Is Never Alone

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FHE enables computation on encrypted data without interaction...

...but someone still needs to encrypt the question and decrypt the answer

**FHE is always a component within a distributed system**



# This Talk: Looking at Encrypted Search

Both Data and Queries are Encrypted

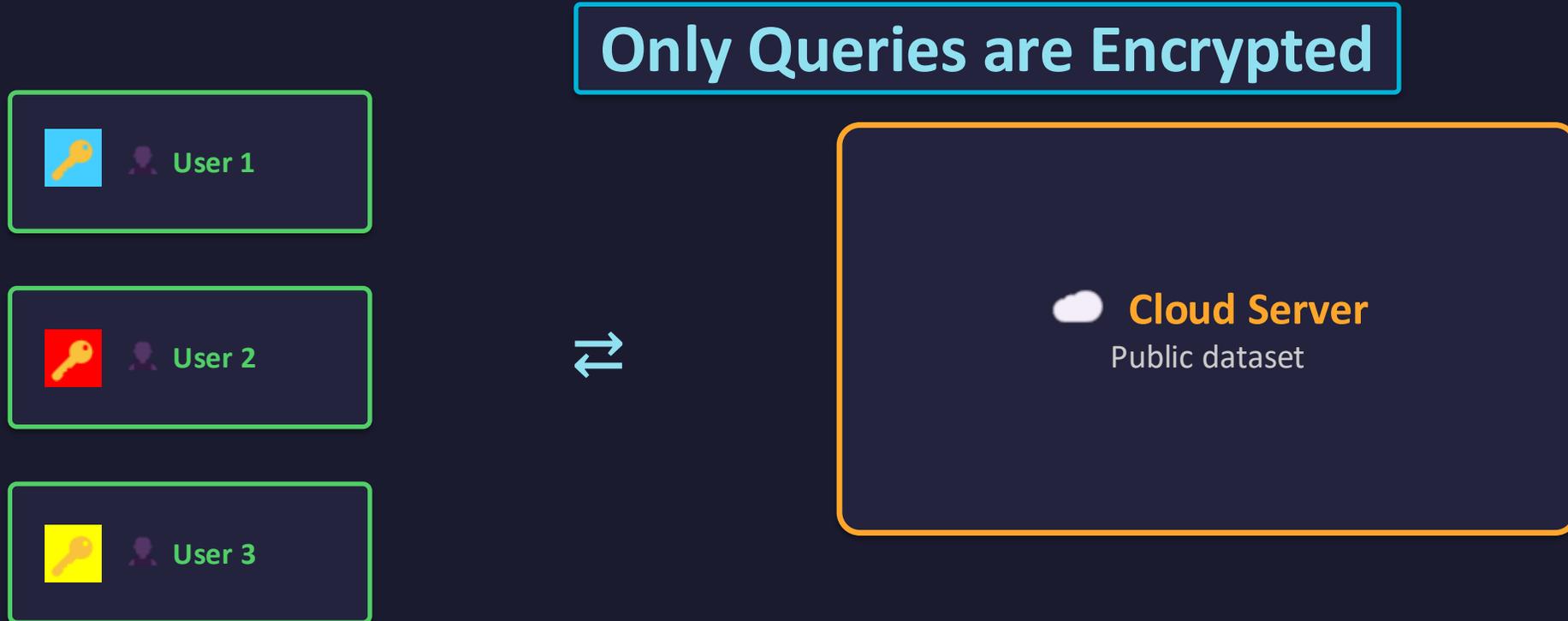


1. Cloud server — stores encrypted dataset, performs computation
2. End-points — many users/devices issuing search queries
3. Key Management in between — at least for decrypting the answers

Similar to Searchable Symmetric Encryption (SSE), but we don't want the leakage

# Aside: A Different Encrypted-Search Problem

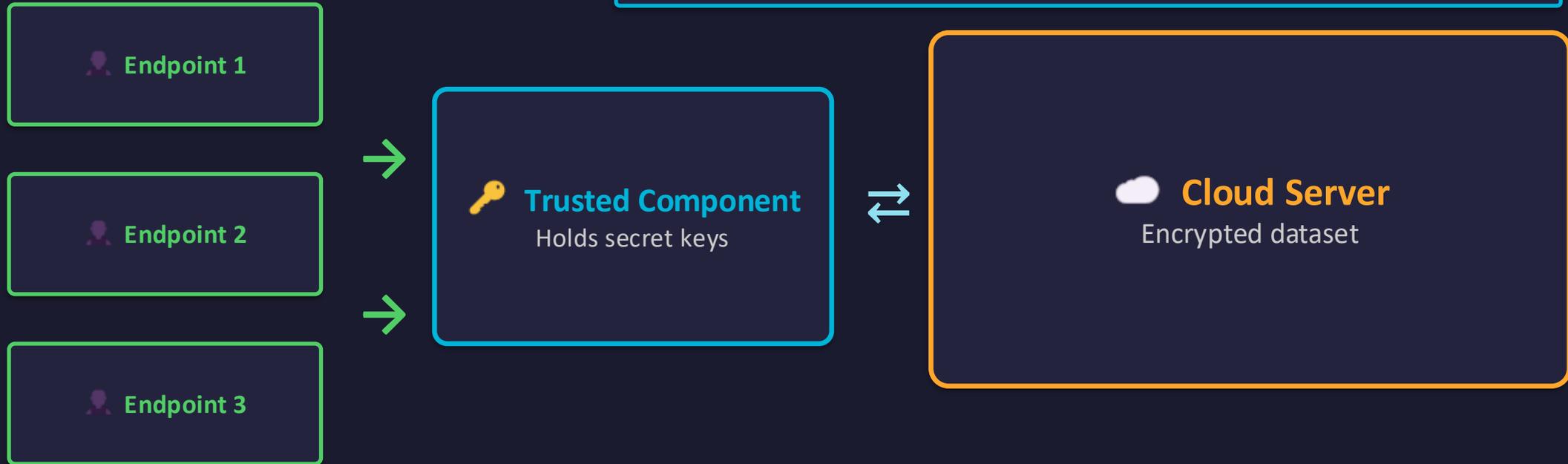
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Also interesting, but not our focus today

# This Talk: Looking at Encrypted Search

Both Data and Queries are Encrypted



1. Cloud server — stores encrypted dataset, performs computation
2. End-points — many users/devices issuing search queries
3. Trusted component — holds secret keys, must be available to the endpoints

# Why Should Anyone Trust This MORE?

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*"You've replaced a cloud that sees my data with a cloud that does encrypted computation plus a trusted component that holds keys. Why is this better?"*

The trusted component sits on my premises? (maybe, but unlikely at scale)

It's simpler, so easier to audit and harden?

It's distributed across multiple nodes?

It's unaware of data semantics?

It's stateless — no evolving state?

It doesn't store the actual data, so breach damage is limited?

Cryptographic solution must block realistic attack vectors that the system faces

# Scenario A — "Keep It Out of the Cloud"

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## Who

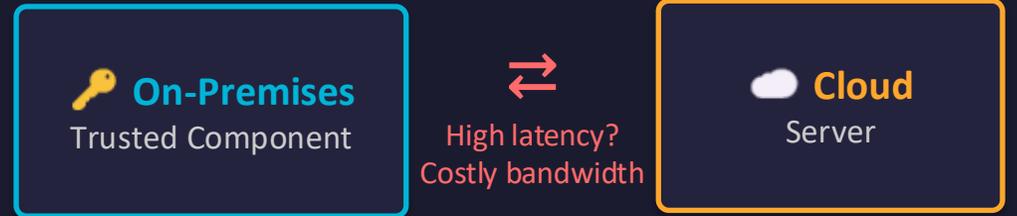
Regulated enterprise (finance, healthcare, government)

## Why

Compliance requires data never leave customer infrastructure in the clear

## Key property

Geographic & administrative separation between keys and data

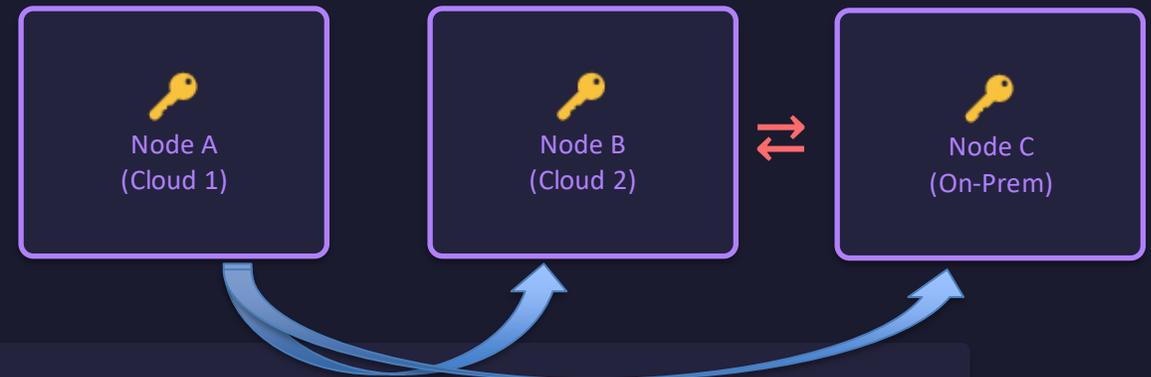


## Implications for technology:

- Bandwidth & round trips are expensive → interactive protocols less appealing
- Favor non-interactive or low-round solutions → FHE/SSE shine here
- Compressing the answer may be needed, but could be computationally expensive

# Scenario B — Distributed Trust (2-out-of-3)

- Eliminates a single point of compromise, maybe multi-cloud?
  - Note: Distributed trusted component can also encrypt (not just decrypt)
- They need to communicate with the server, with each other
  - Maybe then can all be near each other?
- Added Complexity, requires a non-collusion assumption
  - Why is non-collusion reasonable to expect?



- An appealing option: one of these parties deals correlated randomness to the other two

# Scenario C — Reducing the Attack Surface

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## Who

Cloud provider or large platform operator

## Why

Not selling "encrypted search" to customers — using encryption internally to contain blast radius

### Complex Search System

- 10M lines of code
- 200 dependencies
- Daily deploys
- ML models, indices, caches
- Never sees plaintext or keys



### Trusted Component

- Simple, Hardened

*Beneficiary: internal product team, not end customer*

# What Makes "Simple" Simple?

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## Desirable properties of the trusted component:

Small code base — auditable, formally verifiable

Minimal dependencies — small supply-chain surface

Stateless — no evolving state across queries

Not performance critical — can run in a TEE

## Implications for technology:

### ⚠️ FHE decryption in trusted component?

Requires full FHE library — complex?, large?  
Side-channel considerations. Is that "simple"?

➤ Can we get Keygen/Encrypt/Decrypt formally verified?

### ● Information-theoretic protocols

Typically simpler, easier to formally verify.

⚠️ May need correlated randomness (where is it coming from?)

# A Design Space, Not a Single Answer

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**Start with the deployment scenario, not the technology.**

# Let's Get Technical

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A concrete encrypted search problem:

## k-Nearest Neighbors / Search by Similarity

Dataset: N records, each keyed by a vector

Query: a vector

Result: payload of the k records with highest ~~cosine similarity~~ inner product



Useful for  
semantic search,  
biometrics

Two phases, two very different problems:

### Phase 1

Encrypted matrix-vector product (EMVP)  
(Bi)Linear • Massive data • Structured

### Phase 2

Post-processing: extract top-k  
Nonlinear • Smaller data

We'll talk about this part

# Encrypted Matrix-Vector Product

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1. Setup:  $\hat{M} = \text{MatEnc}_{sk}(M)$

# Encrypted Matrix-Vector Product

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1. Setup:  $\hat{M} = \text{MatEnc}_{sk}(M)$
2. Query:  $\hat{q} = \text{QryEnc}_{sk}(q)$

# Encrypted Matrix-Vector Product

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1. Setup:  $\hat{M} = MatEnc_{sk}(M)$
2. Query:  $\hat{q} = QryEnc_{sk}(q)$
3. Answer:  $\hat{a} = Mult(\hat{M}, \hat{q})$

# Encrypted Matrix-Vector Product

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1. Setup:  $\hat{M} = MatEnc_{sk}(M)$
2. Query:  $\hat{q} = QryEnc_{sk}(q)$
3. Answer:  $\hat{a} = Mult(\hat{M}, \hat{q})$
4. Decrypt:  $a = Dec_{sk}(\hat{a}) = M \cdot q^T \in F^N$

# Deployed EMVP for Encrypted Search

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In the context of “proof of humanity”,  $\text{dim} = 10000$

From HE & Garbed-Circuits: *Janus: Safe Biometric Deduplication for Humanitarian Aid Distribution*, EdalatNejad, Lueks, Sukaitis, Narbel, Marelli, Troncoso, S&P 2024

Using 3PC (much faster): *Large-Scale MPC: Scaling Private Iris Code Uniqueness Checks to Millions of Users*, Bloemen, Gillespie, Sippl, Walch, ePrint /2024

# Many Solutions in the Literature

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Using degree-2 HE (duh) [BGN05, Gen09, ...]

Linear HE when the matrix is public [Tiptoe - HDCZ23]

But actually, linear HE is enough also for secret matrix:

– Run public-matrix EMVP with  $M' = M + R$  ( $R = PRG(sk)$ )

• Yields  $a' = M' \cdot q^T$

Client keeps state ( $q$ )

– Client recovers  $M \cdot q^T = a' - R \cdot q^T$

– Can reduce client's work using trapdoored matrix  $R$

[Braverman-Newman 25, Vaikuntanathan-Zamir 25]

• pseudorandom  $R$ , trapdoor used to speed up multiply-by- $R$

• Can be built from LPN, MQ-style assumptions (more if time permits)

# New Approach: EMVP via Secret Dual Codes

[Benhamouda-Chen-H-Ishai-Krawczyk-Mour-Rabin-Rosen 25]

- Symmetric Encryption: Client uses secret key to encrypt dataset, queries (as well as for decryption)
- Field-agnostic: Server/client do matrix-vector products over  $F$ 
  - Somewhat higher-dimension encoded matrix/vectors
- Server work  $\approx Nw$  (see next slide)
- Client work  $o(Nw)$  (using trapdoored matrices, as low as  $\tilde{O}(w)$ )
  - Field-agnostic makes it easy to distributed the client (no noise handling)

# New Approach: EMVP via Secret Dual Codes

## Some variants based on ring-LPN (over- $F$ )

- Everything “optimal upto polylog” (but not concretely efficient)

## Other variants based on new hardness assumption

- Variants of “Learning Subspace with Noise” [Dodis-Kalai-Lovett’09]
- Asymptotic overhead over cleartext as small as  $(1 + o(1))$ 
  - Can get  $< 2x$  for matrix dimensions as small as  $\sim 200$

# Technical Approach: Secret Dual Codes

Pseudorandom codes over  $\mathbb{F}$  determined by secret key  $sk$

*Optionally: structured codes for better efficiency*

## Data Code

Spanned by the rows of  $D \in \mathbb{F}^{w \times n}$

*D derived from  $sk$ , represented by a systematic generating matrix*

$$D = \begin{array}{c} \begin{array}{cc} & n = w + k \\ \begin{array}{|c|c|} \hline I_w & D' \\ \hline \end{array} & \end{array} \end{array} \quad w$$

## Query Code

Spanned by rows of  $C = D^\perp \in \mathbb{F}^{k \times n}$

$$C = \begin{array}{c} \begin{array}{c} n \\ \begin{array}{|c|} \hline D^\perp \\ \hline \end{array} \end{array} \end{array} \quad k$$
$$D \times C^T = \mathbf{0}$$

# EMVP Protocol, First Try

**Setup:**  $\hat{M} = M \times D + R$

$M \in \mathbb{F}^{N \times w}$  — the data matrix (N records, w-dim vectors)

$R$  — pseudorandom matrix determined by sk

$\hat{M}$  is stored at the server

**Query:** Send to server  $\tilde{q}_i = (q_i | \mathbf{0}) + c_i$

$c_i = r_i^T C$  is a random codeword in  $C$

**Answer:** Server returns  $\tilde{a}_i = \hat{M} \cdot \tilde{q}_i^T$

$$\tilde{a}_i = M \cdot q^T + R \cdot \tilde{q}^T$$

**Decryption:** Output  $a = \tilde{a}_i - R \cdot \tilde{q}_i = M q^T$

$$D = \begin{array}{c} \begin{array}{cc} & n = w + k \\ \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \begin{array}{|c|} \hline \\ \hline \end{array} \\ \hline \end{array} \begin{array}{l} I_w \\ D' \end{array} \quad w$$

$$C = \begin{array}{c} \begin{array}{|c|} \hline \\ \hline \end{array} \begin{array}{|c|} \hline \\ \hline \end{array} \quad \begin{array}{l} n \\ D^\perp \end{array} \quad k \\ D \times C^T = \mathbf{0} \end{array}$$

# EMVP Protocol, First Try: Efficiency

**Setup:**  $\hat{M} = M \times D + R$

$M \in \mathbb{F}^{N \times w}$  — the data matrix (N records, w-dim vectors)

$R$  — pseudorandom matrix determined by sk

$\hat{M}$  is stored at the server

Storage overhead:  $n/w = 1 + k/w$

**Query:** Send to server  $\tilde{q}_i = (q_i | \mathbf{0}) + c_i$

$c_i = r_i^T C$  is a random codeword in  $C$

Upload overhead:  $n/w = 1 + k/w$

**Answer:** Server returns  $\tilde{a}_i = \hat{M} \cdot \tilde{q}_i^T$

$$\tilde{a}_i = M \cdot q^T + R \cdot \tilde{q}^T$$

No download overhead

**Decryption:** Output  $a = \tilde{a}_i - R \cdot \tilde{q}_i = M q^T$

# EMVP Protocol, First Try: Security?

Data encryption:  $\hat{M} = M \times D + R$ ,  $R$  is pseudorandom so  $M$  is hidden

Query encryption:  $\tilde{q}_i = (q_i | \mathbf{0}) + r_i^T C$

All the  $q_i$ 's are the same

→ All  $\tilde{q}_i$ 's live in a coset of  $C$

→  $\text{Rank}(\tilde{q}_1, \tilde{q}_2, \dots) \leq k + 1$

If the  $q_i$ 's are random

→  $\text{Rank}(\tilde{q}_1, \tilde{q}_2, \dots) \sim n$

Detectable after enough queries observed!

We want the  $\tilde{q}_i$ 's to be pseudorandom

# EMVP Protocol, With Better Security

Idea: Make  $\tilde{q}_i$  pseudorandom by adding "noise"

- But standard LPN or LWE noise hurts correctness

Instead:

- Mixture noise: w/ prob  $\mu < 1$ , replace  $\tilde{q}_i$  by a sample from a different distribution (e.g., uniform)
  - Repeat/encode-across-columns to correct failures
- Planting noise: reveal a random low-dimensional subspace  $\tilde{Q}_i$  containing  $\tilde{q}_i$

**Query:** Compute  $\tilde{q}_i$  as before. Send to server  $\tilde{Q}_i = [q_{i,1}^T \mid \dots \mid q_{i,s}^T]$   
such that  $\tilde{q}_i = \sum_j \alpha_{i,j} \cdot q_{i,j}$  for random scalars  $\alpha_{i,j}$

**Answer:** Send back  $A_i = \hat{M} \cdot \tilde{Q}_i$

Additional  $s \times$  overhead

**Decrypt:** Output  $a = A_i \cdot (\alpha_{i,1}, \dots, \alpha_{i,s})^T - R \cdot \tilde{q}_i$

Security related to "Learning Subspace with Noise" (LNS), see next slide

# LSN Assumptions (Learning Subspace with Noise)

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Identify "perturbed" vectors from a secret linear/affine subspace

New(ish) type of hardness assumptions:

- Originated for leakage-resilience  
[Dodis–Kalai–Lovett 09]
- Structured variants implicit in doubly-efficient secret-key PIR  
[Boyle–Ishai–Pass–Wootters 17, Canetti–Holmgren–Richelson 17]
- Search-LSN studied by learning theory community  
[Chen–De–Vijayaraghavan 21]
- Usage for PIR, similar to ours [Chen-Ishai-Mour-Rosen 25]
- Equivalent to LPN when  $\mathbb{F}$  is small and  $n = k + 1$

# EMVP Protocol, Improving Efficiency

## Query:

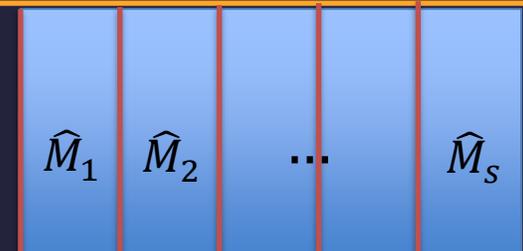
- Compute  $\tilde{q}_i$  as before
- Break  $\tilde{q}_i$  into  $s$  blocks, multiply  $j$ 'th block by a random scalar  $\alpha_{i,j}^{-1}$
- Concatenate and send to server:  $\hat{q}_i = (\hat{q}_{i,1} = \alpha_{i,1}^{-1} \cdot \text{block}_{i,1}, \dots, \hat{q}_{i,s} = \alpha_{i,s}^{-1} \cdot \text{block}_{i,s})$

Upload overhead:  $n/w = 1 + k/w$

$\times \alpha_{i,1}^{-1} \times \alpha_{i,2}^{-1} \dots \times \alpha_{i,s}^{-1}$

## Answer

- Break  $\hat{M}$  into column-blocks  $[\hat{M}_1 | \dots | \hat{M}_s]$
- Return  $A_i = [\hat{M}_1 \cdot \hat{q}_{i,1}^T | \dots | \hat{M}_s \cdot \hat{q}_{i,s}^T]$



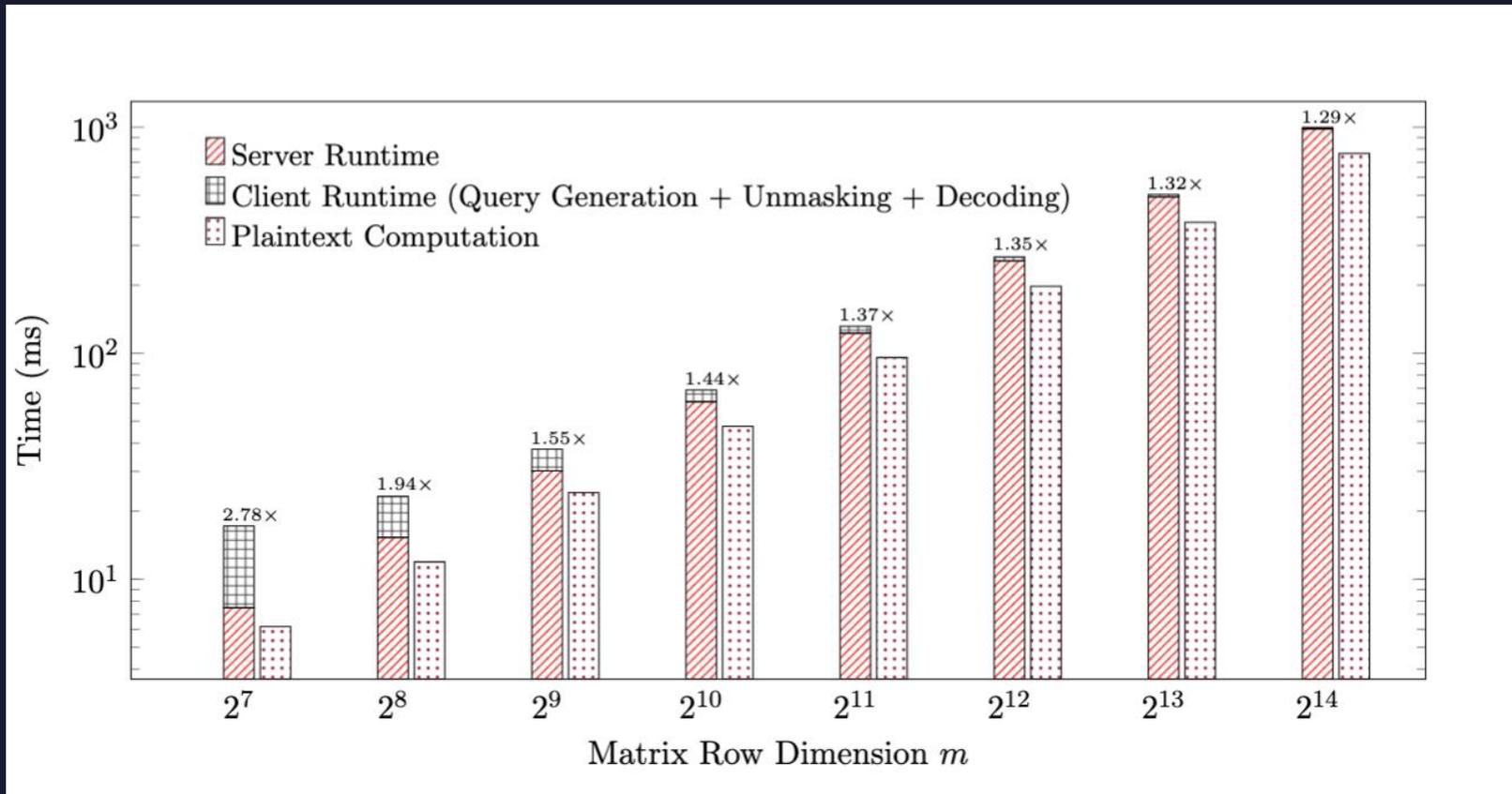
Download overhead  $s \times$

Decrypt: Output  $a = A_i \cdot (\alpha_{i,1}, \dots, \alpha_{i,s})^T - R \cdot \tilde{q}_i$

Client multiplies  $N \times s$  matrix by a vector

# Some Timing Results

- $(w, k, b) = (10000, 2600, 140)$ , over  $Z_p$  with  $|p| = 32$



# Security Under "Split-LSN" Hardness

$$\begin{array}{c} \boxed{r_1} \\ \boxed{r_2} \\ \boxed{r_3} \\ \vdots \end{array} \times \begin{array}{c} \overset{n}{\boxed{C}} \\ \text{Secret code } C \\ \underset{k}{\boxed{\phantom{C}}} \end{array} = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ \vdots \end{array} \quad v_i = r_i \cdot C^T \text{ (broken into blocks of size } b\text{)}$$

$v_1$	$\times \alpha_{1,1}^{-1}$	$\times \alpha_{1,2}^{-1}$	$\dots$	$\times \alpha_{1,s}^{-1}$	$\hat{v}_1$
$v_2$	$\times \alpha_{2,1}^{-1}$	$\times \alpha_{2,2}^{-1}$	$\dots$	$\times \alpha_{2,s}^{-1}$	$\hat{v}_2$
$v_3$	$\times \alpha_{3,1}^{-1}$	$\times \alpha_{3,2}^{-1}$	$\dots$	$\times \alpha_{3,s}^{-1}$	$\hat{v}_3$
					$\vdots$

How hard is it to distinguish the  $v_i$ 's from random?

Algebraic attacks (à la [Raz 09, Arora-Ge 11])  
 $\approx b^{k/b}$  work



# Algebraic Attack

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**Theorem:** For any  $(k, n)$ -code  $C$  and  $d < n - k$ ,  $\exists$  a non-zero degree- $d$  polynomial  $P_C(\cdot)$  s.t.

$$\text{span}(\vec{v}_1, \dots, \vec{v}_d) \cap C \neq \{\vec{0}\} \Rightarrow P_C(\vec{v}_1, \dots, \vec{v}_d) = 0$$

- For us, each block is a vector,  $d = \lceil k/b \rceil + 1$
- The monomials are from a universe of size  $b^d$  (regardless of  $C$ )

## The attack:

- From each  $\hat{q}_i$ , prepare a dimension- $b^d$  vector of potential monomials
$$\vec{u}_i = \text{block}_{i,1} \otimes \text{block}_{i,2} \otimes \dots \otimes \text{block}_{i,d}$$
- The  $\vec{u}_i$ 's are all orthogonal to the coefficient vector of  $P_C$
- So they are rank-deficient, observable after seeing  $b^d$  of them



# Constructing Trapdoored Matrices from LPN

$$R = H \times E$$

- $H$  is a **public code** with fast syndrome, for which LPN holds
- $E$  is a **secret sparse matrix**, derived from the secret key
  - $R$  is pseudorandom under dual-LPN  $(H, He) \approx (H, r)$
- $\vec{w} = R \times \vec{u}$  is fast because:
  - $\vec{v} = E \times \vec{u}$  is fast since  $E$  is very sparse
  - $\vec{w} = H \times \vec{v}$  is fast since  $H$  has fast syndrome

# Even Faster Trapdoored Matrices

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Based on various MQ-style assumptions

$$\boxed{R} = \boxed{S_L} \times \boxed{\Pi_L} \times \boxed{S} \times \boxed{\Pi_L} \times \boxed{S_R}$$

- The  $S$ 's are structured secret matrices
  - E.g.,  $S_L$  multiplies by  $(1, r_L)$  over  $F_{p^k}$
- The  $\Pi$ 's are public permutation matrices
- Speculative (but plausible) new hardness assumptions

# EMVP Protocol: The Moral (1)

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- Super-Efficient “poor man’s FHE”
  - Delegating (secret) linear functions, secrecy almost for free
    - Can be made even more efficient by batching many queries together
  - Can be extended to bilinear forms with very little overhead [[Joshi-Wagner-H-Mishra 26](#)] (used for lexical search)
  - Very easy to authenticate the server’s answer
    - Using homomorphic MACs (a-la-SPDZ)
    - This “verifiable FHE”, malicious security
  - EMVP is “easier than” (does not imply) linear HE
    - Unlikely to even imply CRH/OT/PKE

# EMVP Protocol: The Moral (2)

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- LSN-style assumptions require further study, but...
  - Everything is achievable under LPN with polylog overhead
  - Hiding data relies only on standard assumptions
  - Queries are “locally hidden” information-theoretically

# Back to Encrypted Search

Two phases, two very different problems:

## Phase 1

Encrypted matrix-vector product (EMVP)  
(Bi)Linear • Massive data • Structured

## Phase 2

Post-processing: extract top-k  
Nonlinear • Smaller data

- Can use the new EMVP Protocol
- Before sending the answer, the server holds an encryption of  $M \cdot q^T$ 
  - Much smaller than the full  $M, q$
  - Dimension  $N \times s$  instead  $N \times w$
- Here we need “real FHE”/MPC
- Deep (F)HE to do everything on the server
- Or additive-HE to transform it to additive sharing of the result
  - Then run Client-Server 2PC
  - Very efficient aggregation (OR, Max, Histogram)  
[\[Benhamouda-H-Ishai-Rathee 26\]](#)

# Key Takeaways

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## FHE is never alone

- Thinking of it as part of a larger distributed system may offer simplification options
- Always start from the deployment scenario

## EMVP compression + FHE/MPC post-processing is appealing

- Could be effective in many settings

# and a Reminder

A new FHE benchmarking suite  
<https://fhe-benchmarking.org>

Exactly the encrypted-kNN search that I described here

Please submit your solutions

## A NEW FHE BENCHMARKING SUITE

// FHE\_BENCHMARKING\_V1.0

Andrea Alexandru • Flavio Bergamaschi  
• Shruthi Gorantala • Shai Halevi  
Quality Technologies • Optalysys • Google • AWS  
FHE.org 2026 Conference

### THE PARADIGM SHIFT

**THE GAP:** Potential adopters currently lack a common yardstick. Existing numbers are library-specific, tied to isolated cryptographic micro-benchmarks, or linked to specific academic papers.  
**THE GOAL:** The Suite provides relevant, fully-specified, end-to-end workloads that represent interesting and useful use cases, informing business decisions through performance data standardized across libraries and backends.

### PERSONAS SERVED

- Application Developers  
Assess E2E feasibility and cost of FHE for their workloads.
- FHE Library/Compiler Developers  
Optimize scheme-level performance targets.
- Hardware Vendors  
Guide design and testing of accelerators for FHE workloads.

### METRIC A: WALL-CLOCK

#### Latency & Throughput

Wall-clock time for single workload instances and batch processing performance.

### METRIC B: FOOTPRINT

#### Memory, Storage & Communication

Maximum RAM usage and total storage for keys, ciphertexts, and intermediate values. Bandwidth of data exchange.

### METRIC C: QUALITY METRICS

#### Correctness & Accuracy

Result validation and accuracy loss compared to plaintext reference (where applicable).

### SECURITY MANDATE

#### 128-BIT

Minimum required security level. Submitters must provide formal justification of parameter selection.

### SUITE STRUCTURE

#### HARNES SUBDIRECTORY

Immutable executions and measurement logic provided by organizers.

#### SUBMISSION SUBDIRECTORY

Submitter-filled solution with implementation code and full documentation. Core workload computation must be performed on encrypted data, though pre-processing before encryption and post-processing after decryption are permitted.

### SUBMISSION OPTIONS

- Open-source software  
Complete implementation code.
- Closed-source software  
Pre-compiled libraries or containers.
- Hardware and Backed  
Shims for backend communication. Backends should remain available (for testers) for at least a few weeks following initial submission.

### SUBMISSION PROTOCOL

- Fork Repository [github.com/fhe-benchmarking](https://github.com/fhe-benchmarking) /sw/ subdirectories.
- Implement Workload: populate /submission or /submission-remote subdirectories.
- Maintain Harness: do not modify the /harness subdirectory.
- Document Solution: update the README with all relevant information and optionally provide more documentation in the /docs subdirectory.
- Generate Measurement Files: run the harness with a `--num_runs 3` argument and commit the measurements files.
- Submit Results: make the fork public and notify suite organizers via the form provided at [fhe-benchmarking.github.io](https://fhe-benchmarking.github.io).

### CURRENT WORKLOADS

**VECTOR SEARCH** fetch-by-similarity  
Private database queries using Cosine Similarity search over encrypted data. Essential for private biometrics and RAG.  
DB size: 50K, 100K, 20M  
Record size [b]: 128, 256, 512

**LARGE INTEGER MULTIPLICATION** in-plaintext  
Multiplication of two large encrypted integers. Essential for blockchain applications.  
Batch size: 1, 1K, 100K, 10M  
Size progression [b]: 64, 128, 256

**MACHINE LEARNING INFERENCE** ml-inference  
Privacy-preserving machine learning inference on encrypted inputs. Requirement: must meet accuracy thresholds for batch inference.  
Batch size: 1, 1K, 100K, 1M  
Model progression: MNIST, CIFAR-10, ResNet-50, ..., BERT

### TARGET EVALUATION PLATFORMS

- Software-only submissions are tested on normalized platforms to ensure hardware-agnostic comparisons. Recommended: platforms with 5th-gen Intel Xeon (Emerald Rapids), 96 vCPUs and ample memory.  
EC2 i7ie.24xl GCP c4-highmem-96 Azure Standard-E96s-v6
- Submissions that rely on accelerated computing should specify the acceleration hardware used (e.g., number and type of GPUs).

### DEVELOPMENT ROADMAP

FHE TRANSCIPHERING	ENCRYPTED FACE RECOGNITION	PIR/PSI
PRIVATE SQL QUERIES	FHE SIGNATURES	SPARSE MATRIX MULTIPLICATION

Active contributions from the [homomorphiccryption.org](https://homomorphiccryption.org) community

### FHE BENCHMARKING RESULTS

Example: fetch-by-similarity (Fetch - Small)

Name	Submitter	Envr	Date	Bandwidth			Timing (harness)				Timing (server)	
				Keys	DB	Query	Total	Keygen	DB Enc	Q Enc	Compute	Total
Reference	CPU	2026-02-13 22:21:16	24G	5.6G	24M	1.8759m	4.2226s	33.73s	2.3333ms	1.223m	1.2167m	1.1833m

Get in Touch & Get Involved

[fhe-benchmarking@homomorphiccryption.org](mailto:fhe-benchmarking@homomorphiccryption.org)

[github.com](https://github.com)

[fhe-benchmarking.org](https://fhe-benchmarking.org)

A photograph of a classroom where several students have their hands raised. The focus is on a hand in the foreground, with others blurred in the background. The background is a green chalkboard.

Questions?